Cosmological perturbations for Transient Acceleration Universe

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Introduction

The standard ΛCDM model states that the universe will expand forever at an accelerated form. However recent investigations seem to favor that the currently observed accelerated expansion of the Universe might be a transient phenomenon, implying that a de Sitter phase does not necessarily represent the final state of the cosmic evolution. The model on which the present work relies was developed in [1]. It describes transient cosmological acceleration as the consequence of an interaction between dark matter and dark energy. While the study in [1] was restricted to the homogeneous and isotropic background, this work shows the corresponding perturbation dynamics as well. In particular, we calculate the growth rate of the matter perturbations and the matter power spectrum. We compare our results with the growth-rate data collected as well as those of the WiggleZ survey and with the data from the 2dFGRS program. The results indicate that a transient phase of accelerated expansion is not excluded by current observations.

The Transient Acceleration Model

The dark components of the universe do not conserve separately but interact with each other in such a manner

\[ \rho_m + 3M \rho_m = Q , \quad \rho_x + 3 (1 + \omega) \rho_x = - Q , \]

with \( Q \) being a phenomenological quantity. An interaction modifies the usual behaviour of the matter energy density to \( \rho_m = \rho_m \exp(-f(a)) \) where the function \( f(a) \) encodes the influence of the interactions and \( Q = \rho_m f(a) \). It is convenient to write \( f(a) = 1 + g(a) \).

The following special case is considered

\[ w = -1 , \quad g(a) = \gamma a^2 \exp(-a^2) , \]

where \( \gamma \) is an interaction constant, as a toy model. Under this circumstances the dark-energy density becomes

\[ \rho_x = \rho_x^{0,1} - \gamma a^2 \exp(-a^2) , \]

and

\[ \rho_x^{0,1} = \rho_x - \frac{2}{3} \rho_0 \left[ 1 + \frac{3}{2} \gamma a^2 \right] \exp(-1/a^2) . \]

The interaction re-normalizes the bare (interaction-free) value \( \rho_x^{0,1} \). A transient accelerated is only possible for \( \rho_x^{0,1} = 0 \) since otherwise the constant would always prevail in the long-time limit. This means, for accelerated expansion to be a transient phenomenon, part of the interaction has to cancel the "bare" cosmological constant.

With the requirement \( \rho_x^{0,1} = 0 \) and the definition of the interaction parameter

\[ K = \frac{8 \pi G}{3 H_0^2} \rho_m , \]

one finds that there exists a range

\[ \frac{2 \sqrt[4]{\sigma^2}}{9 \sigma^2 - 4} < K < \frac{2 \sqrt[4]{\sigma^2}}{3 \sigma^2} \]

of possible values for the interaction parameter \( K \) that guarantees an early matter dominated phase, accelerated expansion around the present time and a phase of decelerated expansion in the far-future limit. The corresponding deceleration parameter is shown in Fig. 1. Using the Constitution set, we find best-fit values \( K = 0.018 \) and \( \sigma = 0.12 \) [2].

Newtonian Perturbations

Under conditions that the perturbation have wavelengths much smaller than the Hubble radius, the dynamics is well approximated by a Newtonian analysis.

The perturbed energy balance is, in first order,\n
\[ \delta \dot{h}_{\text{m}} + c_s^2 \delta = - (1 - \beta) \frac{Q}{\dot{\rho}_m} \]

where \( c_s^2 \) is the (non-relativistic) matter velocity and was assumed for simplicity that \( Q = 3Q_{\text{m}}h_0 \) with a \( \beta \) constant that quantifies the perturbed interaction.

The Euler equation in first order reads

\[ \dot{c}_x = \frac{a}{\dot{a}} \ddot{c}_x - \dot{h}_x \]

And the first-order field equation of Newtonian gravity in comoving coordinates

\[ \frac{1}{a^2} \ddot{c}_x = 4\pi G (\rho_m h_0 + \rho_x h_x) \]

We shall assume here \( h_x = \rho_m h_0 \) where \( \alpha \) is a constant that quantifies the dark-energy fluctuation.

With the help of these equations, we can find the basic Newtonian perturbation equation

\[ \ddot{c}_x + \frac{2}{\dot{a}} [1 - \beta] \dot{c}_x = \frac{3}{2} \omega \left( \frac{\rho_m}{\rho_x} \right) + \frac{2}{\dot{a}} \left( \frac{\rho_m}{\rho_x} \right) + \frac{3}{2} \omega \left( \frac{\rho_m}{\rho_x} \right) \]

with

\[ \frac{\rho_m}{\rho_x} = \frac{2}{\dot{a}} + \frac{2}{\dot{a}^2} \]

It is convenient to introduce the growth rate

\[ \frac{d}{d\ln a} \ln \delta \propto \frac{d}{d\ln a} \ln c_x \]

to compare our results with the data sample, among them those of the WiggleZ survey, see Fig. 2, we find best-fit values \( \alpha = -0.36 \) and \( \beta = 1.2 \) [2].

Relativistic Perturbations

We assume that the cosmic medium as a whole can be described by the energy momentum tensor of a perfect fluid \( T_{\mu \nu} = \rho \delta_{\mu \nu} + p g_{\mu \nu} \), where \( \delta_{\mu \nu} = g_{\mu \nu} + \nu_{\mu} \nu_{\nu} \), and \( g_{\mu \nu} \delta_{\mu \nu} = -1 \).

The model can be described as

\[ T^\text{int} = T^\text{int}_m + T^\text{int}_r , \quad T^\text{int}_m = \frac{\rho_m}{\rho_{\text{cr}}} \]

where \( T^\text{int}_m \) represents pressureless (dark) matter and \( T^\text{int}_r \) represents some kind of dark energy, that are assumed to have perfect-fluid structures too.

The case of interacting fluid can be described as \( T^\text{int}_m = Q^\text{int} \) and \( T^\text{int}_r = -Q^\text{int} \). Then perturbed in first-order and consider comoving gauge-invariant quantities, we found the basic relativistic perturbation equation

\[ \delta^{\text{int}} + F(a) \delta^{\text{int}} + G(a) \delta^{\text{int}} = 0 , \]

where

\[ F(a) = \frac{1}{a^2} \left( 1 + \frac{2}{\dot{a}} \right) \left( 1 + \frac{3}{2} \omega \left( \frac{\rho_m}{\rho_x} \right) + \frac{2}{\dot{a}} \left( \frac{\rho_m}{\rho_x} \right) + \frac{3}{2} \omega \left( \frac{\rho_m}{\rho_x} \right) \right) \]

and

\[ G(a) = \frac{1}{a^2} \left( 1 + \frac{2}{\dot{a}} \right) \left( \frac{3}{2} \omega \left( \frac{\rho_m}{\rho_x} \right) \right) \]

where we have assumed the relationship \( \delta_s^\text{int} = \delta^{\text{int}} \) to quantify the dark-energy perturbations in the relativistic level. In the Fig. 3, we display the matter power spectrum, based on the data of the 2dFGRS project, for different values of \( \epsilon \) for \( c_s^2 = 1 \), with a best-fit values \( \epsilon = -0.000023 \) [2].

Conclusions

- A phenomenological model of transient accelerated expansion in which, an interaction in the dark sector, is constitutive for the cosmological dynamic does not seem to contradict current observational data.
- The interaction has to play a twofold role. It has both to cancel a "bare" cosmological constant and, at the same time, to generate a phase of accelerated expansion by itself.
- The perturbation dynamics of the model was investigated both on the Newtonian and on the General Relativity levels.
- A statistical analysis on the basis of the 2dFGRS data revealed that perturbations of the dark-energy component are negligible on scales that are relevant for structure formation. Only on very large scales their contribution might be noticeable.

References