Physics of Cosmic Acceleration

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An experimenter and a theorist go on a hike...
These Lectures are not on Dark Energy

Rene Magritte
Plan of Lectures

1. Cosmic Expansion and Growth
2. Dark Energy as a Field
3. Dark Energy as Gravity
4. Chasing Down Cosmic Acceleration

The first 2/3 of each part will be lecture, the last 1/3 will be questions, discussion, and exercises.
Acceleration is a key element of physics, central to Einstein’s Equivalence Principle.

Gravity = Curvature = Acceleration

Gravity is equivalent to the curvature of spacetime geometry, and determines the motions of particles along geodesics.

Forces (acceleration) change the motions of particles can be viewed as affecting spacetime geometry. Locally, acceleration is equivalent to gravity.
In the presence of gravity or of acceleration, light follows a curved path. Locally, they are equivalent.
The Principle of Equivalence teaches that

**Acceleration = Gravity = Curvature**

Acceleration ⇒ over time will get \( v = \frac{gh}{c} \),
so \( z = \frac{v}{c} = \frac{gh}{c^2} \), (gravitational redshift).

But, \( t' \neq t_0 \) ⇒ parallel lines not parallel (curvature)!
Acceleration has:

- Direct (kinematic) effect on spacetime through \( a(t) \)
- Dynamic effects on objects within spacetime, e.g. growth, ISW

What appears in the metric is the cosmic scale factor \( a(t) \).

\[
ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta \, d\phi^2) \right]
\]

The metric can be spatially flat (\( k=0 \)) but the spacetime is curved if \( \ddot{a} \neq 0 \)

This is exactly the Equivalence Principle:

Gravity = Curvature = Acceleration
Homogeneity and isotropy determine the spacetime to be maximally symmetric and the metric takes the Robertson-Walker form.

\[ ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \]

Spherical symmetry is obvious because the spatial sections involve two-spheres: for constant \( r \) the angular dependence is just

\[ d\omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \]

The key ingredients are

- constant parameter \( k \) – spatial curvature,
- function of time \( a(t) \) – scale (expansion) factor.
k is inverse square radius of curvature, \( k = 1/R_c^2 \).
If \( k = 0 \) then \( R_c = \infty \) and space is flat.
\( k > 0 \) indicates positive curvature (like a sphere),
\( k < 0 \) negative curvature (like a hyperboloid/saddle).

We can also choose to make \( r \) dimensionless (giving dimensions to \( a \)) and normalize \( k = 0, +1, -1 \).
Cosmic Expansion

In front of the spatial part of the metric is the scale factor $a(t)$, scaling all distances. If $a$ increases with time, this indicates cosmic expansion.

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta \, d\phi^2) \right]$$

If $r$ is dimensionful then $a$ is dimensionless and we can normalize $a_{\text{today}} = a_0 = 1$. Cannot simultaneously normalize $k$ and $a$!

2\text{nd} derivatives of the metric $g_{ab}$ form the Ricci tensor, determining spacetime curvature. This is proportional to $\ddot{a}$.
Space flatness: $k=0$

Spacetime flatness: $\ddot{a} = 0$

Exercise 1.1: Show that $\ddot{a} = 0$ is equivalent to a flat (Minkowski) spacetime.

All results coming directly from the metric (spacetime symmetries) are called kinematics.

We have not had to specify any laws of gravity!

Results that require force laws are called dynamics.
Light Propagation

Light signals travel on null geodesics ($ds=0$) and measure $\int dt/a = \int da \ dt/da \ (a/a^2) = \int da^{-1} \ dt/dlna = \int dz/H$. Distances are directly affected by acceleration.

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta \ d\phi^2) \right]$$

If source and observer are comoving, the distance $r_c$ is constant. Thus $\int dt/a = \text{const}$.

Imagine a source pulsing with frequency $\nu \sim 1/dt$. The emission at $t_e + dt_e$ is observed at $t_o + dt_o$. But

$$\int_{t_o}^{t_e} \frac{dt}{a} = \int_{t_o+dt_o}^{t_e+dt_e} \frac{dt}{a} \quad \Rightarrow \quad \frac{dt_e}{a(t_e)} - \frac{dt_o}{a(t_o)} = 0$$
Redshift

Redshift is given by

\[ 1 + z = \frac{\nu_e}{\nu_0} = \frac{\lambda_0}{\lambda_e} = \frac{a_0}{a_e} \]

Note this is a purely kinematic effect.

General formula for redshift is

\[ 1 + z = \frac{(g_{ab}u^a k^b)_e}{(g_{ab}u^a k^b)_0} \]

where \( u^a \) is source 4-velocity, \( k^b \) is photon 4-momentum

Exercise 1.2: What else can affect redshift?
Since acceleration is a property of \( a(t) \), can we detect acceleration directly in redshift?

**Redshifts** are changes in scale/position ("velocities"):  
\[
z = \frac{[a(t_0)-a(t_e)]}{a(t_e)} \rightarrow H_0 (t_0-t_e)
\]

**Redshift shifts** are changes in changes ("acceleration"):  
\[
\frac{dz}{dt_0} = \frac{[\dot{a}_0-\dot{a}_e]}{a_e} = H_0(1+z)-H(z) \rightarrow \Delta z = -zq_0 H_0 \Delta t
\]

**Redshift drift** (Sandage 1962; McVittie 1962; Linder 1991,1997)  
\[
\Delta z = 10^{-8} \text{ over } 100 \text{ years}
\]
# BAO for Acceleration

Acceleration can be seen directly through redshift drift.

\[
\dot{z} = H_0 (1 + z) - H(z)
\]

McVittie/Sandage 1962

Europe wants to build a 40m telescope to stare at quasars for 10 years and measure \( z \) to \( 10^{-10} \).

Instead, use radial BAO of galaxies \( 10^{10} \) years apart.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Equation</th>
<th>Nuisance</th>
<th>Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z ) Drift</td>
<td>( \dot{z}_2 - \dot{z}_1 = H_0 (z_2 - z_1) - (H_2 - H_1) )</td>
<td>( H_0 )</td>
<td>( w &lt; -1/3 )</td>
</tr>
<tr>
<td>radial BAO</td>
<td>( rBAO_2 - rBAO_1 = s(H_2 - H_1) )</td>
<td>( s )</td>
<td>( w &lt; -1 )</td>
</tr>
</tbody>
</table>

**Exercise 1.3:** Show the sign of \( z \) drift gives the sign of acceleration; show the sign of radial BAO gives the sign of \( 1+w \).
Distances are directly affected by acceleration. They are the most practical kinematic way to measure cosmic acceleration.

If we introduce dynamics (forces, interactions) there are many other ways – but we also need to be sure we actually understand the forces, not just the spacetime symmetry.

Direct dynamical detection?
But... Dark energy in solar system = 3 hours of sunlight.

Co-dependence?
Variations of fundamental constants; lab/accelerator/universe (highly model dependent).
What is Dark Energy?

How many dark rectangles do you see?
Beyond Kinematics

Once you go beyond kinematics to dynamics, you have a lot of questions to answer!

What is its dynamics?

Does dark energy interact?

Does dark energy have internal degrees of freedom?

Can we split off matter and radiation?

In these lectures we will mostly assume that dark energy can be treated as a single, independent quantity (so we can talk about matter etc. separately).
Cosmological Framework

Equivalence Principle

→ Metric description of spacetime

Homogeneity and Isotropy

→ Metric is Robertson-Walker

→ Energy-momentum has perfect fluid form \((\rho, p)\)

Gravitational Field Eqs (General Relativity) + Homogeneity and Isotropy

→ Friedmann equations for evolution of spacetime

Equations of State + Friedmann equations

→ Evolution of energy densities
Einstein says gravitating mass depends on energy-momentum tensor: both energy density $\rho$ and pressure $p$, as $\rho + 3p$.

Negative pressure can give negative “mass”

Newton’s 2nd law: Acceleration = Force / mass

$$\ddot{R} = -\frac{GM}{R^2} = -\frac{4\pi}{3}G \rho R$$

Einstein/Friedmann equation:

$$\ddot{a} = -\frac{4\pi}{3}G (\rho + 3p) a$$

Negative pressure can accelerate the expansion
Relation between $\rho$ and $p$ \textit{(equation of state)} is crucial:

\[ w = \frac{p}{\rho} \]

Acceleration possible for $p < -(1/3)\rho$ or $w < -1/3$

What does negative pressure mean?

Consider 1\textsuperscript{st} law of thermodynamics:

\[ dU = -p \, dV \]

But for a spring \[ dU = +k \, xdx \]

or a rubber band \[ dU = +T \, dl \]
Quantum physics predicts that the very structure of the vacuum should act like springs.

Space has a “springiness”, or tension, or vacuum energy with negative pressure.

Review --

*Einstein*: expansion acceleration depends on $\rho + 3p$

*Thermodynamics*: pressure $p$ can be negative

*Quantum Physics*: vacuum energy has negative $p$

Cosmological observations can map the expansion history, measure acceleration, detect vacuum energy.
Equations of motion for the homogeneous background.

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8 \pi G}{3} \rho - k a^{-2}
\]

\[
\frac{\ddot{a}}{a} = - \frac{4 \pi G}{3} (\rho + 3p)
\]

\[
\dot{\rho}_i = -3 \frac{\dot{a}}{a} (\rho_i + p_i)
\]

Only two equations independent because Bianchi identity redundant.
Notation

\[ H(a) = \frac{\dot{a}}{a} \quad q(a) = -\frac{a\ddot{a}}{\dot{a}^2} \quad \Omega_i(a) = \frac{8\pi G \rho_i(a)}{3H^2(a)} \]

\[ \Omega_{\text{tot}}(a) = \sum_i \Omega_i(a) = 1 - \Omega_k(a) = 1 + \frac{k}{a^2 H^2} \]

\[ \rho_i(a) = \rho_i e^{-3 \int_0^a \ln a' \, d \ln a'} [1 + w_i(a')] \sim a^{-3(1+w_i)} \]

\[ \frac{H(a)}{H_0} = \left[ \sum \Omega_i a^{-3(1+w_i)} + 1 - \Omega_{\text{tot}}(a) \right]^{1/2} \]

\[ q(a) = \frac{1}{2} \sum \Omega_i (1+3w_i) a^{-3(1+w_i)}/[H(a)/H_0]^2 \]
Deep enough that is less than 10% energy density? Not next-to-dominant?

Deep enough that have accounted for >2/3 of the acceleration?
Distances relative to low and high redshift have different degeneracies, hence complementarity e.g. Supernovae (R. Kessler) and BAO (Y. Wang)
Equations of motion for linearly perturbed quantities gives growth of structure.

Newtonian approach (doesn’t always work!): Perturb the acceleration equation by

\[ R = R_0 a(t) \left[ 1 - \frac{\delta(t)}{3} \right] \]

that conserves mass

\[ (\rho + \delta \rho) R^3 = \rho R_0^3 a^3(t) \]

This determines growth of density inhomogeneities \( \delta = \delta \rho / \rho \)

\[ \ddot{\delta} + 2H \dot{\delta} - 4\pi \rho \delta = 0 \]

Note expansion (H) slows exponential (Jeans instability) growth to power law in time (\( \delta \sim a \) in matter domination).
Growth $g(a) = (\delta\rho/\rho)/a$ depends purely on the expansion history $H(z)$ – and gravity theory.

$$g'' + \left[ 5 + \frac{1}{2} \frac{d \ln H^2}{d \ln a} \right] g'a^{-1} + \left[ 3 + \frac{1}{2} \frac{d \ln H^2}{d \ln a} - \frac{3}{2} G \Omega_m(a) \right] ga^{-2} = 0$$

Within general relativity ($G = G_N = 1$), expansion determines growth and vice versa.

Acceleration suppresses growth in two ways:

1) the friction term $\sim (3-q)$ so $q<0$ slows growth,

2) the source term $\Omega_m(a)$ is diminished.
Exercise 1.1: Show that $\ddot{a} = 0$ is equivalent to a flat (Minkowski) spacetime.

Exercise 1.2: What else can affect redshift?

Exercise 1.3: Show the sign of $z$ drift gives the sign of acceleration; show the sign of $rBAO$ gives the sign of $1+w$.

For more dark energy resources, see

http://supernova.lbl.gov/~evlinder/scires.html


Mapping the Cosmological Expansion http://arxiv.org/abs/0801.2968

and the references cited therein.