

## Time dependent modeling of galactic cosmic ray transport in the heliosphere

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**Abstract:** We explore the relationship between the galactic cosmic rays (GCRs) intensity changes ( $dI$ ) and the magnitude ( $B$ ) of the interplanetary magnetic field (IMF) for 1996-2012, including positive polarity ( $A > 0$ ) period (1996-2000), maximum of solar activity period with uncertain direction of the sun's polar magnetic field (2000-2003), and negative polarity ( $A < 0$ ) period (2003-2012). The overall inverse linear correlation coefficient ( $cc$ ) between  $dI$  and  $B$  is high, for the full range of the observed 27-day averages of  $B$ , for 1996-2012,  $cc = -0.82 \pm 0.03$ . The breakdown is as follows, for 1996-2000,  $cc = -0.65 \pm 0.07$ , for 2000-2003 the correlation is significantly weaker,  $cc = 0.01 \pm 0.10$ , and for 2003-2012,  $cc = -0.84 \pm 0.05$ . We find that the dependence of  $dI$  on  $B$  may be represented by a power law,  $dI \propto B^\alpha$ . For the full range of the observed 27 day averages of  $B$ , for 1996-2012,  $\alpha = -0.24$ . The breakdown is as follows, for 1996-2000,  $\alpha = -0.19$ ; for 2000-2003,  $\alpha = 0$ , and for 2003-2012,  $\alpha = -0.23$ . We attempt to construct a time-dependent 2-D model of GCR long-term variations taking into account the derived relationship between  $dI$  and  $B$ .

**Keywords:** galactic cosmic rays, time-dependent modeling, 11-year variation

### 1 Introduction

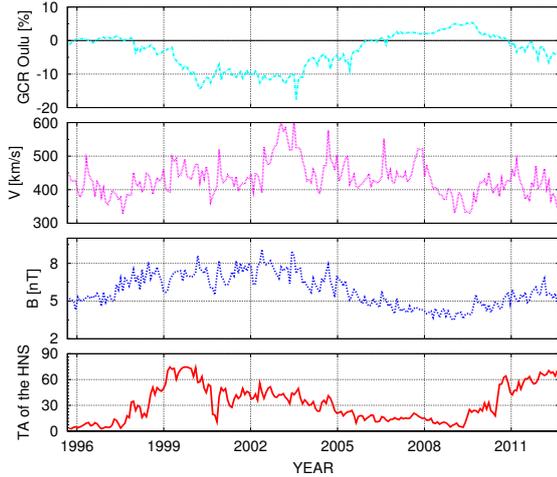
Long period variations of galactic cosmic ray (GCR) intensity has been studied for years using experimental data from different detectors [1]-[9]. Modeling study of GCR transport have been carried out by Ferreira and Potgieter [10]. They used numerical solution of Parker's transport equation to model the modulation of cosmic-ray protons, electrons, and helium for full 11 year and 22 year modulation. Krymsky et al. [11] have constructed a simplified theory for the heliospheric modulation of high-energy cosmic rays in which no adjustable parameters are used. U-soskin et al. [12] have done a series of reconstruction of the monthly values of the modulation potential. Siluszyk et al. [13] developed the 2-D time dependent model of the 11-year variation of the GCR intensity using in situ measurements of solar wind and solar activity parameters; with an acceptable level of compatibility for 1976-1987 (solar cycle 21) when the expected temporal changes of the GCR particle density are shifted for 18 months with respect to the temporal changes of the smoothed experimental data on the GCR intensity. Manuel et al. [14] have calculated time-dependent cosmic ray modulation over multiple solar cycles. Recently Krymsky et al. [15] shown that cosmic rays behave quite differently when even and odd solar cycles alternate. Paouris et al. [16] investigate galactic cosmic ray modulation in relation to solar activity indices and heliospheric parameters during the years 1996-2010 covering solar cycle 23. Chowdhury et al. [17] show that GCR experience various types of modulation from different solar activity features and influence space weather and the terrestrial climate. There are a few interesting and very much attractive studies by various authors, but 4 pages space of this paper confine us to mention many of other authors.

Despite significant advances in our understanding of many aspects of the long period variation of GCR intensity, a single parameter or group of parameters that completely describes the 11-year cycles of GCR intensity has not been found. It should be underlined that the theoretical modeling of the long period variation of the GCR intensity also remains complex because of the uncertainty selecting of the suitable parameters to be implemented in the transport equation. Nevertheless, only one criterion remains for selecting an appropriate set of parameters, the degree of compatibility of modeling results with the experimental data. So, a composite study of the experimental data and modeling is much valuable. Our aim in this paper is to find a relationships among cosmic ray intensity variations for period of 1996-2012 and parameters of solar wind and solar activity, and then use these parameters in transport equation to model a behavior of GCR intensity registered by neutron monitors (NMs).

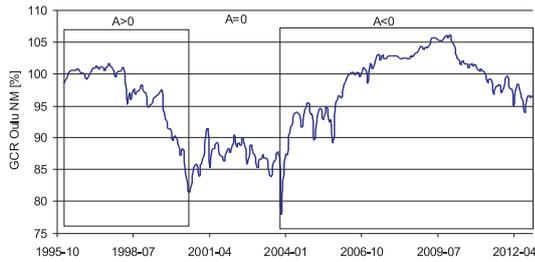
### 2 Experimental Data

To find a relationships among cosmic ray intensity variations and parameters of solar wind and solar activity, we use the GCR intensity for Oulu NM, solar wind velocity  $V$ , magnitude  $B$  of the interplanetary magnetic field (IMF) and tilt angle  $TA$  of the heliospheric neutral sheet (HNS) for 1996-2012. In figure 1 are presented changes of the 27-day averages of the GCR intensity for Oulu NM (top panel), solar wind velocity  $V$ , magnitude  $B$  of the IMF and tilt angle  $TA$  of the HNS for 1996-2012.

In order to set up a relationship between changes of the GCR intensity and magnitude  $B$  of the IMF, intensity of GCR for Oulu NM was recalculated to free space using the temporal changes of the rigidity spectrum index  $\gamma$  found for long period variations of GCR intensity [18]. The re-



**Fig. 1:** Temporal changes of the 27-day averages of the GCR intensity for Oulu NM, solar wind velocity  $V$ , magnitude  $B$  of the IMF and tilt angle  $TA$  of the HNS for 1996-2012.

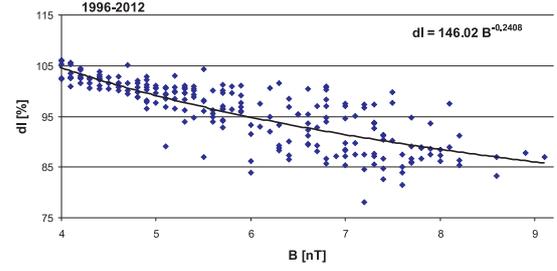


**Fig. 2:** Temporal changes of the 27-day averages of the GCR intensity in free space for Oulu NM for 1996-2012 .

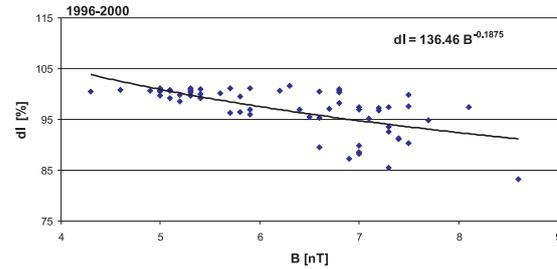
calculated intensity of the GCR intensity for Oulu NM to free space is presented in figure 2; it corresponds to the changes of GCR intensity for 10 – 15GV. To find a power law dependence of the GCR intensity  $dI$  (Fig. 2) and magnitude  $B$  of the IMF ( $dI \propto B^\alpha$ ) for 1996-2012, scattering diagram is presented in figure 3. A power law approximation  $dI \propto B^\alpha$  shows that  $\alpha = -0.24$ . Taking into account that minimum epochs of solar activity can be characterized as drift dominated periods, but maximum-as a diffusion dominated, the whole period 1996-2012 is divided as follows, 1996-2000 ( $A > 0$ ), 2000-2003 ( $A = 0$ ), and 2003-2012 ( $A < 0$ ). Scatter plots of the 27-day averages of GCR intensity changes  $dI$  for Oulu NM and magnitude  $B$  of the IMF according the breakdown are presented in figures 4-6. The breakdown is as follows, for 1996-2000,  $\alpha = -0.19$  (figure 4); for 2000-2003,  $\alpha = 0$  (figure 5), and for 2003-2012,  $\alpha = -0.23$  (figure 6).

### 3 Model

In this paper we model the long period variation of the GCR during years 1996-2012 (the Bartel's rotation (BR) periods 2219-2447). The proposed model of the long period variations of the GCR intensity is based on the Parker's time-dependent transport equation [19]:



**Fig. 3:** Scatter plot of the 27-day averages of GCR intensity changes  $dI$  for Oulu NM and magnitude  $B$  of the IMF for 1996-2012.



**Fig. 4:** Scatter plot of the 27-day averages of GCR intensity  $dI$  for Oulu NM and IMF  $B$  for the positive polarity ( $A > 0$ , 1996-2000).

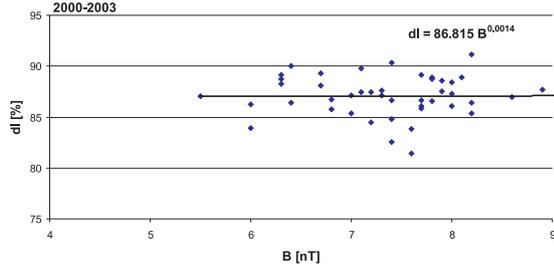
$$\frac{\partial N}{\partial \tau} = \nabla \cdot (K_{ij}^S \cdot \nabla N) - (v_d + U) \cdot \nabla N + \frac{1}{3} \frac{\partial}{\partial R} (NR) \nabla U \quad (1)$$

Where  $N$  and  $R$  are omnidirectional distribution function and rigidity of GCR particles, respectively;  $\tau$  - time,  $U$  solar wind velocity,  $v_d$  is the drift velocity. We set up the dimensionless density  $f = \frac{N}{N_0}$ , time  $t = \tau \cdot \tau_0$  and distance  $r = \frac{\rho}{\rho_0}$ ; where  $N_0$  is density in the Local Interstellar Medium (LISM),  $\rho$  and  $\rho_0$  are the radial distance and size of the modulation region;  $\tau_0$  is the characteristic time corresponding to the changes in heliosphere for the certain class of GCR variation. For the considered model we accept that  $\tau_0$  is equal to 218 BR. A size of the modulation region  $\rho_0 = 100AU$  and the upwind-downwind asymmetry of the heliosphere is not taken into account, as far  $\rho_0$  is significantly greater (more than 10 times) than the larmor radius of GCR particles to which NMs respond. It would be proper to underline here, that Krymsky [20] and Parker [19] independently established the crucial roles of diffusion, convection and expanding solar wind in modulation of GCR; also, they showed that diffusion coefficient of GCR has the tensor character in the interplanetary space.

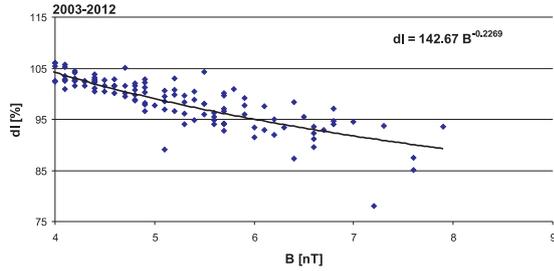
The equation (1) for the dimensionless variables  $f$ ,  $R$  and  $t$  in the 2-D spherical coordinate system can be written:

$$A_0 \frac{\partial f}{\partial t} = A_1 \frac{\partial^2 f}{\partial r^2} + A_2 \frac{\partial^2 f}{\partial \theta^2} + A_3 \frac{\partial^2 f}{\partial r \partial \theta} + A_4 \frac{\partial f}{\partial r} + A_5 \frac{\partial f}{\partial \theta} + A_6 f + A_7 \frac{\partial f}{\partial R} \quad (2)$$

where  $A_0 = \frac{\rho_0^2}{\tau_0 K_{ij}}$  and the coefficients  $A_1, A_2, \dots, A_7$  are functions of the spherical coordinates  $(r, \theta)$ , rigidity  $R$  of the GCR particles and the time  $t$  (detailed coefficients are



**Fig. 5:** Scatter plot of the 27-day averages of GCR intensity  $dl$  for Oulu NM and IMF  $B$  for the maximum conditions of solar activity, period (2000-2003).

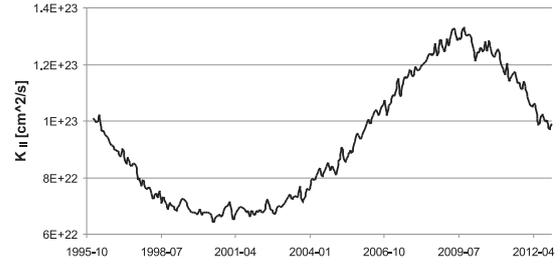


**Fig. 6:** Scatter plot of the 27-day averages of GCR intensity  $dl$  for Oulu NM and IMF  $B$  for the negative polarity period ( $A < 0$ , 2003-2012).

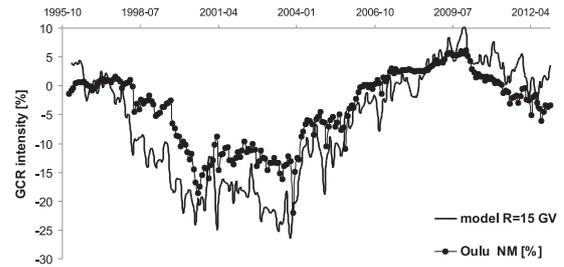
given in [13]). The anisotropic diffusion tensor of GCR  $K_{ij} = K_{ij}^{(S)} + K_{ij}^{(A)}$  consists of the symmetric  $K_{ij}^{(S)}$  and  $K_{ij}^{(A)}$  antisymmetric parts. We implement a drift velocity of GCR particles in a model as [21],  $\langle v_{d,i} \rangle = \frac{\partial K_{ij}^{(A)}}{\partial x_j}$ . This expression is equivalent to the standard formula for  $\langle v_d \rangle$  [22]. The IMF vector  $B$  is written, as [23, 24]:

$$B = (1 - 2H(\theta - \theta'))(B_r e_r) \quad (3)$$

where  $H$  is the Heaviside step function changing the sign of the polar magnetic field in each hemisphere and  $\theta'$  corresponds to the heliolatitudinal position of the HNS,  $e_r$  is the unite vector directed along the component  $B_r$  of the IMF of the two dimensional Parker field [25]. Parker's spiral heliospheric magnetic field is implemented through the angle  $\psi = \text{Arctan} \frac{-B_\phi}{B_r} = \text{Arctan} \frac{\Omega r \sin \theta}{U}$  in anisotropic diffusion tensor of GCR particles ( $\psi$  is the angle between magnetic field lines and radial direction in the equatorial plane) and ratios  $\beta = \frac{K_\perp}{K_{||}}$  and  $\beta_1 = \frac{K_d}{K_{||}}$  of the perpendicular  $K_\perp$  and drift  $K_d$  diffusion coefficients to the parallel  $K_{||}$  diffusion coefficient:  $\beta = \frac{1}{1+(\omega\tau)^2}$  and  $\beta_1 = \frac{\omega\tau}{1+(\omega\tau)^2}$  where  $\omega\tau = 300B\lambda cR^{-1}$ , the magnitude  $B$  of the IMF equals 5.5 nT in minimum epoch and 8.5 nT in maximum epoch of solar activity;  $\lambda$  linearly depends on the rigidity  $R$  of GCR particles and  $c$  is the speed of light. So, for the cosmic ray particles of rigidity  $R > 10GV$  the perpendicular  $K_\perp$  and drift  $K_d$  diffusion coefficients are proportional to the parallel  $K_{||}$  diffusion coefficient. The billiard ball diffusion is not generally the best approximation [26]-[29] but it works well at high rigidities to which NM and muon telescopes are respond,  $R > 10 - 15GV$  [30, 31]. A parallel diffusion coefficient used in modeling is expressed, as:



**Fig. 7:** The changes of the diffusion coefficient given by eq. (4) for GCR particles with rigidity 15GV.



**Fig. 8:** GCR intensity variation for Oulu NM and model computations for 1996-2012.

$$K_{||} = K_0 K(r) K(B) K(t) K(R, \gamma(t, R)) \quad (4)$$

where  $K_0 = 3.12 \cdot 10^{19} \text{ cm}^2/\text{s}$ ,  $K(r) = 1 + 0.5r/r_0$ ;  $K(B) = B^\alpha$ , as obtained based on the experimental data analysis, for 1996-2000,  $\alpha = -0.19$ ; for 2000-2003,  $\alpha = 0$ , and for 2003-2012,  $\alpha = -0.23$ ;  $K(R, \gamma(t, R)) = R^{\gamma(t, R)}$ , where  $\gamma(t, R) = (2.55t^4 - 0.98t^3 - 3.44t^2 + 1.89t + 0.76) \cdot 0.7R^{0.1}$  contributes to the changes of the parallel diffusion coefficient  $K_{||}$  due to dependence on the GCR particles rigidity  $R$ ;  $K(t) = \text{Exp}(4.5(2 - \gamma(t, R)))$ , function  $K(t)$  is introduced to make a consistent changes of diffusion coefficient  $K_{||}$  throughout the 11 year cycle of solar activity. The exponent  $\gamma(t, R)$  reflects the changes of the variational rigidity spectrum in the modulation of the GCR particles. According to the experimental data analysis (e.g., [18], and references therein) this exponent increases in the maximum of solar activity. In this paper we assume that  $\gamma(t, R)$  reflects this changes i.e. in minimum of solar activity in 1996  $\gamma(t, R) \approx 0.7$ , in the maximum increases up to  $\gamma(t, R) \approx 1$ , then decreases to  $\gamma(t, R) \approx 0.6$  in 2009, and then increase again in ascending phase of the 24th solar activity cycle. In this way the expression  $R^{\gamma(t, R)}$  is larger in maxima epoch than in the minima epoch. On the other hand diffusion coefficient  $K_{||}$  should be greater in minimum epoch than in maximum epoch of solar activity. It is adjustable by the changes of a rate of the function  $K(t)$  in the manner that the product of  $K(t)K(R, \gamma(t, R))$  should diminish from minimum (1996) to maximum (2000) epochs of solar activity and then increase again (Fig. 7).

In the model are implemented the 27-day averages (BR 2219-2447) of the in situ data of the interplanetary magnetic field magnitude  $B$ , solar wind velocity  $V$  and the heliospheric tilt angle  $TA$  of the HNS (Fig. 1). Also, in the model is implemented the waviness of the HNS (for details see [13]). The neutral sheet drift was taken into account ac-

ording to the boundary condition method [23], when the delta function at the HNS is a consequence of the abrupt change in sign of the IMF. Drift effect due to gradient and curvature of the regular IMF is implemented in the model by means of the ratio of the drift  $K_d$  diffusion coefficients to the parallel  $K_{II}$  diffusion coefficient,  $\beta_1 = \frac{K_d}{K_{II}}$ . We consider that a drift effect is scaled to zero during the reversal time (2000-2003) of the Sun's global magnetic field (diffusion dominated period). The equation (2) was transformed to the algebraic system of equations using the implicit finite difference scheme, the details of the numerical solution of the eq. (2) are given in [32].

Results of the numerical solution of the equation (2) for rigidity  $R = 15GV$  and changes of the GCR intensity observed by Oulu NM are presented in figure 8. Figure 8 shows that in general there is some agreement between experimental data for Oulu NM and modeling results dealing mainly with shape and average amplitude of the 11-year variations of the GCR intensity. It is hardly expected that any short period fluctuations of the GCR intensity could be completely explained by modeling. We show by this modeling that an implementation of in situ measurements of the solar wind and solar activity parameters makes model more compatible with the experimental data; also, an inclusion of the real (observed) power law dependence of the GCR variations on the strength  $B$  of the IMF, different in various epochs of solar and magnetic activities, makes the proposed model more realistic.

## 4 Conclusions

1. We show that there is different overall inverse linear relationship between changes of the GCR intensity  $dI$  and strength  $B$  of the IMF in various phases of solar magnetic activity. For positive and negative polarity periods coefficient correlations are high; for 1996-2000 ( $A > 0$ ),  $cc = 0.65 \pm 0.07$ , for 2003-2012 ( $A < 0$ ),  $cc = -0.84 \pm 0.05$ , while for maximum epoch 2000-2003,  $cc = 0.01 \pm 0.10$ . It is clearly seen that in maximum epoch of solar activity diffusion is completely dominated, and role of  $B$  is minimal.
2. We show that the dependence of  $dI$  on  $B$  can be represented by a power law,  $dI \propto B^\alpha$  with significant oddity in maximum epoch comparing with positive and negative polarity epochs. Namely,  $\alpha = 0$ , while for  $A > 0$ ,  $\alpha = -0.19$ , and  $\alpha = -0.23$ , for  $A < 0$ .
3. We compose 2-D time dependent model included in situ measurements of  $B$ , solar wind velocity  $V$ , and tilt angle  $TA$ . The finding dependence between  $dI$  and  $B$  is taken into account in parallel diffusion coefficient.
4. In general there is a good agreement between experimental data of Oulu NM and modeling results dealing mainly with shape and average amplitude of the 11-year variations of the GCR intensity. We conclude that an inclusion of the real (observed) power law dependence of the GCR variations  $dI$  on the strength  $B$  of the IMF in various epochs of solar magnetic activities, makes the proposed model more realistic. However, a short period fluctuations of the GCR intensity could not be completely reproduced by the proposed model.

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## References

- [1] I. V. Dorman and L. I. Dorman, J. Geophys. Res., 72, (1967) 1513
- [2] J. A. Lockwood and W. R. Webber, J. Geophys. Res., 101, A10, (1996) 21573-21580
- [3] Fujii, Z., and F. B. McDonald, J. Geophys. Res., 102, A11, (1997) 24201-24208
- [4] I.G. Usoskin, H. Kananen, G.A. Kovaltsov, K. Mursula, P. Tanskanen, J. Geophys. Res., 103, A5, (1998) 9567-9574
- [5] G.A. Bazilevskaya, A.E. Svirzhevskaya, Space Sci. Rev., 85, (1998) 431-521
- [6] G.F. Krymsky, P.A. Krivoschapkin, V.P. Mamrukova, S.K. Gerasimova, Proc. 29th ICRC, 2, (2005) 231-234
- [7] R. T. Gushchina, A.V. Belov, Vladimir N. Obridko, Berta D. Shelting, Proceedings of the 21st ECRS, Kosice, (2008) 226-230
- [8] M.V. Alania, K. Iskra, M. Siluszyk, Adv. Space Res., 45, 10, (2010) 1203-1210
- [9] H.S. Ahluwalia, R.C. Ygbuhay, C. Lopate, R.C. Ygbuhay and M.L. Duldig, Adv Space Res., 46, 17, (2010) 934-941
- [10] S. E. S. Ferreira, M. S. Potgieter, Astrophys. J., 603, (2004) 744-752
- [11] G. F. Krymsky et al., Journal of Experimental and Theoretical Physics, 104, 2, (2007) 189-195
- [12] I. G. Usoskin, G. A. Bazilevskaya and G. A. Kovaltsov, J. Geophys. Res., 116, (2011) A02104, doi:10.1029/2010JA01610
- [13] M. Siluszyk, A. Wawrzynczak, M.V. Alania, Journal of Atmospheric and Solar-Terrestrial Physics, 73, 13, (2011) 1923-1929, doi:10.1016/j.jastp.2011.05.003
- [14] Manuel et al., Advances in Space Research, 47 (2011) 1529-1537
- [15] G. F. Krymsky et al., Astronomy Letters, 2012, 38, 9, (2012) 609-612
- [16] E. Paouris, H. Mavromichalaki, A. Belov, R. Gushchina, V. Yanke, Solar Phys. 280, (2012) 255-271
- [17] P. Chowdhury, K. Kudela, B. N. Dwivedi, Solar Physics, 286, 2, (2013) 577-591
- [18] M.V. Alania, K. Iskra, M. Siluszyk, Adv. Space Res., 41, 2, (2008) 267-274
- [19] E.N. Parker, Planet. Space Sci., 13, (1965) 9-49.
- [20] G. F. Krymsky, Geomagnetism and Aeronomy, 4, (1964) 763-769.
- [21] J. R. Jokipii, E. H. Levy, W. B. Hubbard, Astrophysical Journal, 1, 213, (1977) 861-868.
- [22] B. Rossi, S. Olbert, Introduction to the physics of space, New York: McGraw-Hill, 1970
- [23] J. R. Jokipii, D.A. Kopriva, Astrophys. J., 234, (1979) 384-392.
- [24] J. Kota, J. R. Jokipii, Astrophys. J., 1, 265, (1983) 573-581.
- [25] E. N. Parker, Astrophys. J., 128, (1958) 664-676.
- [26] S. Parhi, R. A. Burger, J. W. Bieber, W. H. Matthaeus, Proceedings of the 27th ICRC, (2001) 3670-3673.
- [27] S. Parhi, J. W. Bieber, W. H. Matthaeus, R. A. Burger, The Astrophysical Journal, 585, 1, (2003) 502-515.
- [28] S. Parhi, J. W. Bieber, W. H. Matthaeus, R. A. Burger, Journal of Geophysical Research, 109, A1, (2004) A01109.
- [29] A. Shalchi, G. Li, G. P. Zank, Astrophysics and Space Science, 325, 1, (2010) 99-111.
- [30] J.R. Jokipii, Rev. of Geophysics and Space Physics, 9, (1971) 27-87.
- [31] A. Shalchi and R. Schlickeiser, Astrophys. J., 604, (2004) 861-873.
- [32] A. Wawrzynczak, M.V. Alania, Lecture Notes in Computer Science, Part I, LNCS 6067, Springer-Verlag Berlin Heidelberg (2010) 105-114.