Measuring Large-Scale Anisotropy in the Arrival Directions of Cosmic Rays Detected at the Telescope Array and the Pierre Auger Observatory above 10^{19} eV

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Abstract: Spherical harmonic moments are well-suited for characterizing anisotropy in the flux of cosmic rays. So far, above 10^{19} eV, no study has revealed a significant departure from isotropy. The dipole vector and the quadrupole tensor are of special interest, and access to the full set of multipoles could provide essential information for understanding the origin of ultra-high energy cosmic rays. Full-sky coverage allows the measurement of the spherical harmonic coefficients in an unambiguous way. This can be achieved by combining data from observatories located in both the northern and southern hemispheres. In this work, we present the prospects for a combined analysis using data recorded at the Telescope Array and the Pierre Auger Observatory. The main challenges are to account adequately for the relative exposures of both experiments and possibly different absolute energy normalizations. Using Monte-Carlo simulations, we show how these challenges will be addressed in an empirical way and illustrate the expected sensitivity of the methodology for the present observatory exposures.

Keywords: Telescope Array, Pierre Auger Observatory, Ultra-High Energy Cosmic Rays, Large-Scale Anisotropies, Full-Sky Coverage.

The large-scale distribution of arrival directions of cosmic rays is an important observable in attempts to understand their origin. This is because this observable is closely connected to both their source distribution and their propagation. Due to the scattering in magnetic fields, the anisotropy imprinted in the arrival directions is mainly expected at large scales up to the highest energies. Large-scale patterns with anisotropy contrast at the level of $10^{-4}-10^{-3}$ have been reported by several experiments for energies below $\simeq 10^{15}$ eV where the high intensity of cosmic rays allows the collection of a large number of events. For energies above $\simeq 10^{15}$ eV, the decrease of the intensity with energy makes it more challenging to collect the statistics required to reveal amplitudes at the percent level which might be expected, in particular above $\simeq 10^{18}$ eV.

The anisotropy of any angular distribution on the sphere is encoded in the corresponding set of spherical harmonic moments $a_{\ell m}$. The dipole vector and the quadrupole tensor are of special interest, but an access to the full set of multipoles is relevant to characterise departures from isotropy at all scales. However, since cosmic ray observatories at ground level have only a partial-sky coverage, the recovering of these multipoles turns out to be nearly impossible without explicit assumptions on the shape of the angular distribution. In most cases, only the dipole (combination of a_{1m} coefficients) and the quadrupole (combination of a_{2m} coefficients) moments can be estimated with a sensible resolution under the assumption that the flux of cosmic rays is purely dipolar or purely dipolar and quadrupolar, respectively. Evading such hypotheses and thus measuring the multipoles to any order in an unambiguous way requires full-sky coverage. Full-sky coverage can only be achieved through the meta-analysis of data recorded by observatories located in both hemispheres of the Earth.

The Telescope Array, located in the Northern hemisphere (mean latitude $+39.3°$), and the Pierre Auger Observatory, located in the Southern hemisphere (mean latitude $-35.2°$), are the two largest experiments ever built dedicated to the study of ultra-high energy cosmic rays. Given the respective latitudes of both observatories, and given that the data sets from the Telescope Array and the Pierre Auger Observatory to be potentially combined consist of events with zenith angle up to $55°$ and $60°$ respectively, full-sky coverage can be indeed achieved. The present report is aimed at designing and studying in detail an effective way to combine data sets from both experiments while keeping under control the directional exposure. In the concern to facilitate this first joint analysis, the foreseen energy threshold, 10^{19} eV, is chosen to guarantee that both surface detector arrays operate with full detection efficiency for any of the local angles selected in each data set [1, 2]. This guarantees that each exposure function should follow purely geometric expectations to a high level. The main challenge in combining both data sets is to account adequately for the relative exposures of both experiments. In addition, since there are numerous sources of detector-dependent systematic uncertainties in the determination of the energy of a cosmic ray primary, there is presumably a difference in the energy scale between both experiments. Such a potential shift in energy leads to different counting rates above some fixed energy threshold, which induces fake anisotropies. Formally, these fake anisotropies are similar to the ones resulting from a shift in the relative exposures of the experiments, except in the case of energy-dependent anisotropies in the underlying flux of cosmic rays.

The observed angular distribution of cosmic rays, $dN/d\Omega$, can be naturally modeled as the sum of Dirac functions on the surface of the unit sphere whose arguments are the arrival directions $\{\mathbf{n}_1, ..., \mathbf{n}_N\}$ of the events. Throughout the paper, arrival directions are expressed in the equatorial coordinate system (declination δ and right

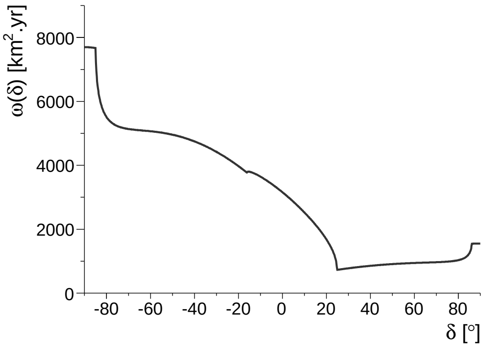

Figure 1: Total directional exposure above 10^{19} eV as obtained by summing the nominal individual ones of the Telescope Array and the Pierre Auger Observatory, as a function of the declination.

ascension α) since this is the most natural one tied to the Earth to describe the directional exposure of any experiment. The random sample $\{\mathbf{n}_1, ..., \mathbf{n}_N\}$ results from a Poisson process whose average is the flux of cosmic rays $\Phi(\mathbf{n})$ coupled to the directional exposure $\omega(\mathbf{n})$ of the considered experiment :

$$\left\langle \frac{dN(\mathbf{n})}{d\Omega} \right\rangle = \omega(\mathbf{n})\Phi(\mathbf{n}). \quad (1)$$

As any angular distribution on the unit sphere, the flux of cosmic rays $\Phi(\mathbf{n})$ can be decomposed in terms of a multipolar expansion onto the spherical harmonics $Y_{\ell m}(\mathbf{n})$:

$$\Phi(\mathbf{n}) = \sum_{\ell \geq 0} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\mathbf{n}). \quad (2)$$

Any anisotropy fingerprint is encoded in the $a_{\ell m}$ multipoles. Non-zero amplitudes in the ℓ modes arise from variations of the flux on an angular scale $\simeq 1/\ell$ radians.

The directional exposure of each observatory provides the effective time-integrated collecting area for a flux from each direction of the sky. In principle, the combined directional exposure of the two experiments should be simply the sum of the individual ones. However, individual exposures have here to be re-weighted by some empirical factor b due to the unavoidable uncertainty in the relative exposures of the experiments. The parameter b can be viewed as a fudge factor which absorbs any kind of systematic uncertainties in the relative exposures, whatever the sources of these uncertainties. This empirical factor is arbitrarily chosen to re-weight the directional exposure of the Pierre Auger Observatory relative to the one of the Telescope Array :

$$\omega(\mathbf{n}; b) = \omega_{\mathrm{TA}}(\mathbf{n}) + b\,\omega_{\mathrm{Auger}}(\mathbf{n}). \quad (3)$$

Dead times of detectors modulate the directional exposure of each experiment in sidereal time and therefore in right ascension. However, once averaged over several years of data taking, the relative modulations of both ω_{TA} and ω_{Auger} in right ascension turn out to be not larger than few thousandths, yielding to non-uniformities in the observed angular distribution at the corresponding level. Given that the limited statistics currently available above 10^{19} eV cannot allow an estimation of each $a_{\ell m}$ coefficient with a precision better than a few percent, the non-uniformities of ω_{TA} and ω_{Auger} in right ascension can be neglected so that both functions are considered to depend only on the declination hereafter. On the other hand, since the high energy threshold guarantees that both experiments are fully efficient in their respective zenithal range $[0-\theta_{\max}]$, the dependence on declination is purely geometric [3] :

$$\omega_i(\mathbf{n}) = A_i \left(\cos\lambda_i \cos\delta \sin\alpha_m + \alpha_m \sin\lambda_i \sin\delta \right), \quad (4)$$

where λ_i is the latitude of the considered experiment, the parameter α_m is given by

$$\alpha_m = \begin{cases} 0 & \text{if } \xi > 1, \\ \pi & \text{if } \xi < -1, \\ \arccos\xi & \text{otherwise,} \end{cases} \quad (5)$$

with $\xi \equiv (\cos\theta_{\max} - \sin\lambda_i \sin\delta)/\cos\lambda_i \cos\delta$, and the normalisation factors A_i are tuned such that the integration of each ω_i function over 4π matches the (total) exposure of the corresponding experiment. For $b = 1$, the resulting $\omega(\delta)$ function is shown in figure 1.

In practice, only an estimation $\overline{b}$ of the factor b can be obtained, so that only an estimation of the directional exposure $\overline{\omega}(\mathbf{n}) \equiv \omega(\mathbf{n}; \overline{b})$ can be achieved through equation 3. The procedure used for obtaining $\overline{b}$ from the joint data set will be described below. The resulting uncertainties propagate into uncertainties in the measured $\overline{a}_{\ell m}$ anisotropy parameters, in addition to the ones caused by the Poisson nature of the sampling process when the function ω is known exactly.

With full-sky but non-uniform coverage, the customary recipe for decoupling directional exposure effects from anisotropy ones consists in weighting the observed angular distribution by the inverse of the *relative* directional exposure function :

$$\frac{d\tilde{N}(\mathbf{n})}{d\Omega} = \frac{1}{\overline{\omega}_r(\mathbf{n})} \frac{dN(\mathbf{n})}{d\Omega}. \quad (6)$$

The relative directional exposure is the dimensionless function normalized to unity at its maximum. When the function ω (or ω_r) is known from a single experiment, the averaged angular distribution $\langle d\tilde{N}/d\Omega \rangle$ is, from equation 1, identified with the flux of cosmic rays $\Phi(\mathrm{n})$ times the total exposure of the experiment. Due to the finite resolution to estimate b, the relationship between $\langle d\tilde{N}/d\Omega \rangle$ and $\Phi(\mathrm{n})$ is here not any longer so straightforward :

$$\left\langle \frac{d\tilde{N}(\mathbf{n})}{d\Omega} \right\rangle = \left\langle \frac{1}{\overline{\omega}_r(\mathbf{n})} \right\rangle \omega(\mathbf{n})\Phi(\mathbf{n}). \quad (7)$$

However, for an unbiased estimator of b with a resolution better than $\simeq 10\%$ (the actual resolution on b will be shown hereafter to be of the order of $\simeq 3.5\%$), the relative differences between $\langle 1/\overline{\omega}_r(\mathbf{n}) \rangle$ and $1/\omega_r(\mathbf{n})$ are actually smaller than 10^{-3} in such a way that $\langle d\tilde{N}/d\Omega \rangle$ can still be identified to $\Phi(\mathbf{n})$ times the total exposure to a high level. Consequently, the recovered $\overline{a}_{\ell m}$ coefficients defined as

$$\overline{a}_{\ell m} = \int_{4\pi} d\Omega \frac{d\tilde{N}(\mathbf{n})}{d\Omega} Y_{\ell m}(\mathbf{n}) = \sum_{i=1}^{N} \frac{Y_{\ell m}(\mathbf{n}_i)}{\overline{\omega}_r(\mathbf{n}_i)} \quad (8)$$

provide unbiased estimators of the underlying $a_{\ell m}$ multipoles since the relationship $\langle \overline{a}_{\ell m} \rangle = a_{\ell m}$ can be established by propagating equation 7 into $\langle \overline{a}_{\ell m} \rangle$.

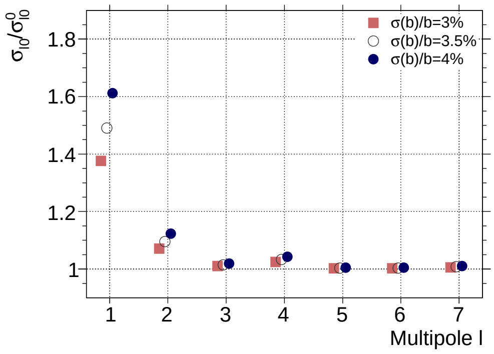

Figure 2: Illustration of the dependence of the resolution on the recovered $\overline{a}_{\ell 0}$ coefficients upon the uncertainty on b, for different values of the resolution on b. On the $y-$axis, the term $\sigma_{\ell 0}^0$ is obtained by dropping the second term inside the square root in the expression of $\sigma_{\ell 0}$ (see equation 9).

Using the estimators defined in equation 8, the expected resolution $\sigma_{\ell m}$ on each $a_{\ell m}$ multipole can be inferred by propagating the second moment of $d\tilde{N}/d\Omega$ into the covariance matrix of the estimated $\overline{a}_{\ell m}$ coefficients accordingly to the Poisson statistics. In the case of relatively small $\{a_{\ell m}\}_{\ell\geq 1}$ coefficients compared to a_{00}, this leads to :

$$\sigma_{\ell m} \simeq \frac{a_{00}}{\sqrt{4\pi}}\left[\frac{\sqrt{4\pi}}{a_{00}}\int_{4\pi} d\Omega \left\langle \frac{1}{\overline{\omega}_r^2(\mathbf{n})}\right\rangle \omega(\mathbf{n}) Y_{\ell m}^2(\mathbf{n}) + \int_{4\pi} d\Omega d\Omega' \left[\left\langle \frac{1}{\overline{\omega}_r(\mathbf{n})\overline{\omega}_r(\mathbf{n}')}\right\rangle \omega(\mathbf{n})\omega(\mathbf{n}') - 1\right] Y_{\ell m}(\mathbf{n}) Y_{\ell m}(\mathbf{n}')\right]^{1/2}. \tag{9}$$

If b were known with perfect accuracy, the second term in equation 9 would vanish, and the resolution of the $a_{\ell m}$'s would be similar to that for a single experiment. The second term adds the effect of the uncertainty in the relative exposures of the two experiments. For a directional exposure independent of the right ascension, it is non-zero for $m = 0$ only, as expected. Its influence is illustrated in figure 2, where the ratio between the total expression of $\sigma_{\ell 0}$ and the partial one ignoring this second term inside the square root is plotted as a function of the multipole ℓ for different resolution values on b. While this ratio amounts to $\simeq 1.5$ for $\ell = 1$ and $\sigma(b)/b = 3.5\%$, it falls to $\simeq 1.1$ for $\ell = 2$ and then tends to 1 for higher multipole values. Consequently, in accordance with naive expectations, the uncertainty on the b factor mainly impacts the resolution on the dipole coefficient a_{10} while it has a small influence on the quadrupole coefficient a_{20} and a marginal one on higher order moments $\{a_{\ell 0}\}_{\ell\geq 3}$.

The hybrid nature of both observatories enables the assignation of the energy of each event to be derived in a calorimetric way through the calibration of the shower size measured with the SD arrays by the energy measured with the fluorescence telescopes on a subset of high quality hybrid events [4, 5]. Nevertheless, though the techniques are nearly the same, there are differences as to how the primary energies are derived at the Telescope Array and the Pierre Auger Observatory. Currently, systematic uncertainties in the energy scale of both experiments amount to about 20% and 14% respectively [6, 7]. Uncovering and understanding the sources of systematic uncertainties in the respective energy scales is out of the scope of this report and will be addressed elsewhere (see for instance [8]). Rather, the aim pursued here is only to guarantee that the relative exposures between both observatories is kept under control in an accurate way, whatever the unknown differences in energy scale. To this end, a purely empirical cross-calibration procedure exploiting the overlapping part of the sky exposed to both experiments is designed for estimating *in fine* reliable anisotropy parameters.

A band of declinations around the equatorial plane is exposed to the fields of view of both experiments, namely for declinations between $-15°$ and $25°$. This overlapping region can be used for *cross-calibrating empirically* the energy scales and for delivering an overall unbiased estimation of the $a_{\ell m}$ multipoles in the case of isotropy. Though the cross-calibration of the energy scale is not a mandatory input for the procedure, it constitutes however a reasonable starting point for studying anisotropies in the arrival directions of all events detected in excess of roughly the same energy threshold by both experiments. The procedure is based on an iterative algorithm which is now detailed. Considering as a first approximation the flux $\Phi(\mathbf{n})$ as isotropic, and given N_{TA} events observed in total above 10^{19} eV, the number of events N_{Auger} corresponding to that particular energy threshold can be inferred accordingly to a simple proportionality :

$$N_{\mathrm{Auger}} = \frac{\int_{4\pi} d\Omega\ \omega_{\mathrm{Auger}}(\mathbf{n})}{\int_{4\pi} d\Omega\ \omega_{\mathrm{TA}}(\mathbf{n})} N_{\mathrm{TA}}. \tag{10}$$

The energy threshold guaranteeing equal intensities for both experiments is then provided by selecting the N_{Auger} highest energy events to be combined with the N_{TA} events. The resulting joint data set consists then of all events with energies in excess of 10^{19} eV in terms of the energy scale used at the Telescope Array. Using the joint data set built in this way, a first estimation of b can be made by counting the ΔN_{TA} and $\Delta N_{\mathrm{Auger}}$ observed in the overlapping region $\Delta\Omega$:

$$\overline{b}^{(0)} = \frac{\Delta N_{\mathrm{Auger}}}{\Delta N_{\mathrm{TA}}} \frac{\int_{\Delta\Omega} d\Omega\ \omega_{\mathrm{TA}}(\mathbf{n})}{\int_{\Delta\Omega} d\Omega\ \omega_{\mathrm{Auger}}(\mathbf{n})}. \tag{11}$$

Inserting $\overline{b}^{(0)}$ into $\overline{\omega}$, 'zero-order' $\overline{a}_{\ell m}^{(0)}$ coefficients can be obtained. This set of coefficients is only a rough estimation, due to the limiting assumption on the flux (isotropy).

On the other hand, the expected number of events in the common band for each observatory, $\Delta n_{\mathrm{TA}}^{\mathrm{exp}}$ and $\Delta n_{\mathrm{Auger}}^{\mathrm{exp}}$, can be expressed from the underlying flux $\Phi(\mathbf{n})$ and the true value of b as :

$$\begin{aligned} \Delta n_{\mathrm{TA}}^{\mathrm{exp}} &= \int_{\Delta\Omega} d\Omega\ \Phi(\mathbf{n})\omega_{\mathrm{TA}}(\mathbf{n}) \\ \Delta n_{\mathrm{Auger}}^{\mathrm{exp}} &= b\int_{\Delta\Omega} d\Omega\ \Phi(\mathbf{n})\omega_{\mathrm{Auger}}(\mathbf{n}). \end{aligned} \tag{12}$$

From equations 12, and from the set of $\overline{a}_{\ell m}^{(0)}$ coefficients, an iterative procedure estimating at the same time b and the set of $a_{\ell m}$ coefficients can be considered in the following way :

$$\overline{b}^{(k+1)} = \frac{\Delta N_{\mathrm{Auger}}}{\Delta N_{\mathrm{TA}}} \frac{\int_{\Delta\Omega} d\Omega\ \overline{\Phi}^{(k)}(\mathbf{n})\omega_{\mathrm{TA}}(\mathbf{n})}{\int_{\Delta\Omega} d\Omega\ \overline{\Phi}^{(k)}(\mathbf{n})\omega_{\mathrm{Auger}}(\mathbf{n})}, \tag{13}$$

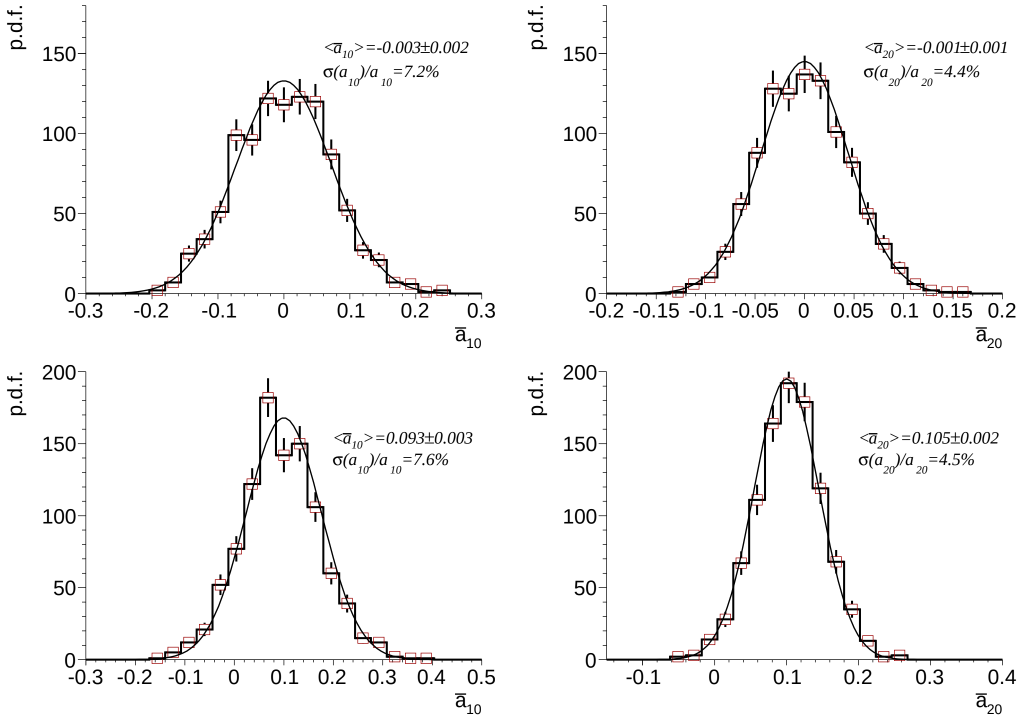

Figure 3: Reconstruction of a_{10} (left) and a_{20} (right) through the iterative procedure, in the case of an underlying isotropic flux (top) or of an anisotropic input flux $\Phi(\mathbf{n}) \propto 1+0.1Y_{10}(\mathbf{n})+0.1Y_{20}(\mathbf{n})$ (bottom). Expectations are shown as the Gaussian curves with resolution parameters as in equation 9.

where $\Delta N_{\mathrm{Auger}}$ and ΔN_{TA} as derived in the first step are used to estimate $\Delta n_{\mathrm{TA}}^{\mathrm{exp}}$ and $\Delta n_{\mathrm{Auger}}^{\mathrm{exp}}$ respectively, and $\overline{\Phi}^{(k)}$ is the flux estimated with the set of $\overline{a}_{\ell m}^{(k)}$ coefficients.

Whether this iterative procedure delivers *in fine* unbiased estimations of the set of $a_{\ell m}$ coefficients with a resolution given by equation 9 can be tested by Monte-Carlo simulations. For 1,000 mock samples with a number of events similar to the one of the actual joint data set and with ingredients corresponding to the actual figures in terms of total and directional exposures, the distributions of reconstructed low orders $\overline{a}_{10}$ and $\overline{a}_{20}$ multipoles are shown in figure 3 (top panels) after $k=10$ iterations in the case of an underlying isotropic flux of cosmic rays. A systematic shift of 20% in energy scale is simulated in each sample; while the directional exposures used in equation 10 correspond to the ones used for generating the events. The averages of the reconstructed histograms are found in agreement with expectations; while, taking as input the observed RMS of the distribution of $\overline{b}$ ($\simeq 3.5\%$) and assuming Gaussian functions, the RMS of the $\overline{a}_{\ell m}$ distributions are found in agreement with equation 9. In practice, these results are found to be stable as soon as $k=4$.

With the exactly same ingredients, the simulations can be used to test the procedure with an underlying anisotropic flux of cosmic rays, chosen here such that $\Phi(\mathbf{n}) \propto 1+0.1Y_{10}(\mathbf{n})+0.1Y_{20}(\mathbf{n})$. Results of the Monte-Carlo simulations are shown in figure 3 (bottom panels) for the specific a_{10} and a_{20} coefficients. In the case of an underlying anisotropic flux of cosmic rays, it is important to note that deciding upon a fixed number of events N_{Auger} through an equation valid in the case of isotropy only (equation 10) is expected to imprint some fake pattern that cannot be fully absorbed. The resulting biases on the $\overline{a}_{10}$ and $\overline{a}_{20}$ are however small, as evidenced in figure 3 (bottom panels).

The cross-calibration procedure designed in this study makes it possible to use the powerful multipolar analysis method for characterising the sky map of ultra-high energy cosmic rays. It pertains to any full-sky coverage achieved by combining data sets from different observatories, and opens a rich field of anisotropy studies. This technique will be applied to data sets from the Telescope Array and the Pierre Auger Observatory and will be reported in a near future.

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