33nd International Cosmic Ray Conference, Rio de Janeiro 2013
The Astroparticle Physics Conference

ICRC 2013

Determining an average value of the absolute air-fluorescence yield

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Abstract: An average value of the available measurements of the absolute air-fluorescence yield is presented. The impact of several factors on the results reported by the experiments, in particular the evaluation of the energy deposition, is discussed. As a result of this analysis, we obtain an absolute value of the fluorescence yield for the 337 nm band in dry air at 800 hPa and 293 K of 7.06 ± 0.25 photons/MeV. This result is in full agreement with that recently published by the Airfly Collaboration. The average calculated here uses the results provided by the whole community working in this field.

Keywords: air-fluorescence yield, fluorescence telescopes, energy deposition, Monte Carlo.

1 Introduction

The air-fluorescence yield FY, that is, the number of fluorescence photons induced by charged particles (mostly electrons and positrons) per unit of energy deposited in air, is a key parameter for the precise determination of the primary energy of cosmic rays using the fluorescence technique.

The FY is a function of wavelength and also depends on atmospheric parameters (i.e., pressure, temperature and humidity) through collisional quenching. The most convenient procedure for the analysis of cosmic-ray data is to combine an accurate measurement of the absolute yield for either a given molecular band or a wavelength interval of interest with the relative intensities of the fluorescence spectrum and those parameters related to atmospheric dependencies. Note that the uncertainty in the absolute value translates almost linearly to the energy scale of the fluorescence telescopes.

Several measurements of the absolute yield are available in the literature [1, 2, 3, 4, 5, 6, 7, 8]. Both HiRes and TA use the measurement of Kakimoto et al. [1] for the 300 - 400 nm interval, while Auger has been using the result at the 337 nm band of Nagano et al. The Airfly Collaboration has recently published a very accurate measurement of the FY of the 337 nm band (4% uncertainty) [8] that is going to replace that of Nagano et al. in the analysis of the Auger data [9].

In this paper, we calculate an average absolute value of the FY for the 337 nm band. To determine the associated uncertainty, a critical analysis of several factors of the above measurements has been carried out. In particular, the influence of the determination of the energy deposition on the FY has been carefully studied. A non-negligible fraction of the energy lost by the incident beam in a typical collision chamber is spread by secondary electrons that leave the field of view of the detector, and therefore, do not contribute to the observed fluorescence [10]. This effect has been ignored by some authors [1, 2, 3], resulting in an underestimation of the FY. Implications of this systematic error on the cosmic-ray spectrum at the highest energies were discussed by Nagano [11].

A detailed MC algorithm that simulates individual interactions of electrons was developed to calculate the energy deposition in air for any geometry [10]. The results of these simulations were used in [12] to apply corrections to some FY values, and a preliminary average of the FY was presented in [13]. The algorithm has been upgraded, finding an excellent agreement within 1% with Geant4 [14]. The corresponding updated analysis, which includes the recent measurements of Airfly and Dandl et al. [7], is shown here.

Our MC algorithm can also calculate the number of emitted fluorescence photons for electrons incident in a given air volume. Combining this result with the energy deposition, a theoretical prediction of the absolute FY is obtained, which will be compared with the experimental ones.

2 Overview of available measurements

Most experiments use electrons from either a ^{90}Sr radioactive source [1, 2, 3, 4, 6], with an average energy around 1 MeV, or accelerators [1, 4, 5], which can provide energies in the GeV range. On the other hand, Dandl et al. employed an electron gun producing a dc-beam of ~ 10 keV electrons, and Airfly used 120 GeV protons from the FNAL Test Facility. The FY is expected to be basically independent of the type and energy of the incident particle. However, our MC algorithm predicts a weak energy dependence as a consequence of differences in the energy spectrum of secondary electrons [10]. Similar studies that we are presently carrying out for protons show a FY almost identical to that for electrons with same velocity (paper in preparation).

Narrow-band filters were used in some experiments [1, 2, 6, 7, 8], providing the yield for the reference 337 nm band. Therefore, these results can be compared directly. On the other hand, some measurements [1, 3, 4, 5] were performed using wide-band filters similar to those employed in fluorescence telescopes, typically collecting light in the 300 - 400 nm spectral range. For these measurements, a wavelength normalization has been made to obtain the FY for the 337 nm band (section 3).

The determination of the energy deposited in the field of view of the detector is required to determine the FY, except for the particular case of the experiment of Dandl et al., where electrons of 10 keV stop within a volume of about 1 mm^3. In recent experiments [4, 5, 6, 8], the energy deposition was carefully evaluated by means of detailed Geant4 or EGS4 simulations. However, in other well-known works [1, 2, 3], the energy deposition was assumed to be equal to the electron energy loss calculated from the Bethe-Bloch formula, i.e., neglecting the contribution of secondary electrons that escape the detector field of view. Note that this assumption only affects the evaluation of the deposited energy, not the measurement of the fluorescence intensity. Therefore, the results of these experiments should be considered fully reliable if appropriate corrections on the energy deposition are applied. For this purpose, simulations of these experiments have been performed, with an estimated uncertainty of 2% in the energy deposition [14]. Most experiments, however, neglect this error contribution in the FY.

Some additional aspects of these measurements (e.g., effects due to differences in the air composition) have been taken into consideration for the evaluation of the uncertainty in the average FY. A comprehensive study of all these factors will be published elsewhere.

3 Normalization and corrections

Measurements of the yield integrated over a wide wavelength interval were normalized to the 337 nm band using the experimental relative intensities of Airfly [15] as well as our theoretical relationships given in [10]. Results were normalized to common conditions (i.e., 800 hPa, 293 K) using either the pressure and temperature dependence reported by the authors or the precise quenching data from [15]. Nevertheless, results are very weakly dependent on the quenching parameters at typical experimental conditions. Details on the normalization procedure are given in [12].

As pointed out above, some results have been corrected to account for the effect of secondary electrons escaping the detector field of view. The measurement of Kakimoto et al. at 1.4 MeV is increased by 10%, that of Nagano et al. at 0.85 MeV is increased by 11% and those of Lefeuvre et al. at 1.1 MeV and 1.5 MeV by 9% and 11%, respectively. On the other hand, large corrections, ranging from +22% to +25%, were applied to the measurements of Kakimoto et al. for electrons of 300 - 1000 MeV.[1]

In figure 1, results normalized to 800 hPa and 293 K (dry air) and with the just above-mentioned corrections are plotted against the electron energy. An electron-equivalent energy of 60 MeV was assigned to the measurement of Airfly for 120 GeV protons according to the scaling laws for the Bethe-Bloch stopping power. Measurements are consistent with a FY independent of the electron energy (dashed line). The solid line represents the theoretical absolute yield calculated from our simulation for electrons crossing a sphere of 5 cm radius filled with air at 800 hPa and 293 K. According to these predictions, the FY is constant at energies larger than 100 MeV while it increases by 2.5% at 1 MeV and by 10% at 1 keV. Note that this weak energy dependence is also compatible with the experimental data[2]

For the purpose of determining the average, the final result reported by each experiment was used instead of the individual measurements shown in figure 1. For instance, the ‘official’ result of Kakimoto et al. is that at 1.4 MeV, for which the effect of secondary electrons is of 10%.

The normalized FY values used to compute the average are shown in figure 2. The original results of Kakimoto et al., Nagano et al. and Lefeuvre et al. are also shown in the figure (grey bars) to illustrate the impact of our corrections. The blue bar represents the theoretical absolute yield predicted by our MC algorithm (100 MeV electrons), with an estimated uncertainty of 20% – 25% (preliminary).

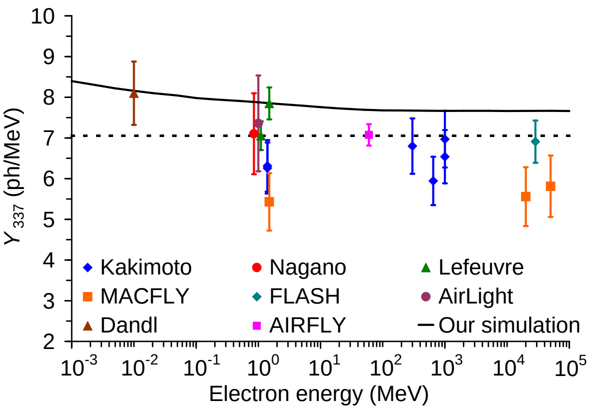

Fig. 1: Comparison of available FY measurements normalized to common conditions (337 nm band, 800 hPa and 293 K) and with the appropriate corrections to account for the effects of secondary electrons neglected by some authors. The horizontal dashed line at 7.06 ph/MeV shows our average FY determined in section 4. The solid line represents the theoretical prediction of the absolute yield from our MC simulation. See text for details.

4 Procedure to determine the average

The average has been computed using the following expression:

$$\langle Y \rangle = \frac{\sum_i Y_i/\sigma_i^2}{\sum_i 1/\sigma_i^2}\,, \tag{1}$$

where Y_i and σ_i are the normalized FY value and uncertainty of experiment i. An average of 7.06 ph/MeV is obtained from the data sample shown in figure 2. This result is represented by the solid vertical line in the figure.

If the experimental uncertainties were assumed to represent the actual standard deviations of the corresponding (normal) probability distributions, the above weighted mean would be the best estimator of the FY and its uncertainty $\sigma_{\langle Y \rangle}$ would be given by

$$\sigma_{\langle Y \rangle}^2 = \frac{1}{\sum_i 1/\sigma_i^2}\,. \tag{2}$$

On the other hand, we note that the quoted experimental uncertainties may be underestimated in some cases. To be more conservative, the uncertainty of the average is calculated instead from

$$\sigma_{\langle Y \rangle}^2 = \frac{\max\left(1, \chi^2/\mathrm{ndf}\right)}{\sum_i 1/\sigma_i^2}\,, \tag{3}$$

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[1] The corrections basically rely on the electron energy, while they are very weakly dependent on the geometrical details of the collision chamber at near atmospheric conditions. See [12] for details.

[2] The contribution of the energy released by an extensive air shower by electrons with energy smaller than 1 MeV is only of about 22% [16]. Therefore, this weak dependence of the FY on the electron energy has no significant impact on the calibration of the fluorescence telescopes.

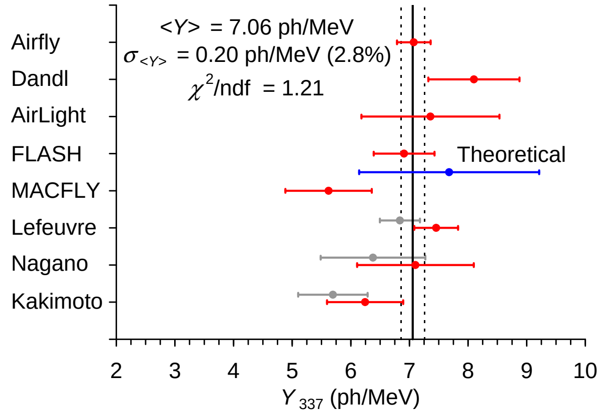

Fig. 2: Graphical representation of the normalized FY values and uncertainties used to compute the average. The original results of Kakimoto et al. [1], Nagano et al. [2] and Lefeuvre et al. [3] are also shown (grey bars) to illustrate the impact of our corrections. The weighted average (1) and associated uncertainty (3) are represented by solid and dashed vertical lines. Note that a more conservative uncertainty is given in section 5. The blue bar represents the theoretical absolute yield predicted by our MC algorithm.

with

$$\chi^2/\mathrm{ndf} = \frac{1}{n-1}\sum_i \frac{(Y_i - \langle Y \rangle)^2}{\sigma_i^2}\,, \qquad (4)$$

where ndf stands for the number of degrees of freedom and $n = 8$ is the size of the data sample. These expressions yield $\chi^2/\mathrm{ndf} = 1.21$ and an uncertainty of 2.8%, which is represented by the dashed vertical lines in figure 2. We think, however, that this uncertainty is still underestimated. Discussion of results and a more conservative estimation of the uncertainty in the average are given in the next section.

5 Discussion of results

Several remarks can be drawn from the above results. There is a good consistency of measurements, as shown by the fact that the χ^2/ndf parameter is close to unity. The effect on the average of removing any single measurement from the sample is less than ±1.7%, and the effect of removing any two measurements is less than ±3.9%. In all cases, differences in the average (relative to the full sample) are lower than the uncertainty obtained from (3) for the corresponding partial sample, which further proves the consistency of results and suggests that measurements are not correlated.

The corrections to the measurements of Kakimoto et al., Nagano et al. and Lefeuvre et al. do eliminate a bias due to the above-mentioned effect of secondary electrons escaping the detector field of view. If our corrections were not included, an average value of 6.82 ph/MeV would be obtained, which is lower by an amount of 3.3%. Note that this bias is unacceptable if we aim at reaching an uncertainty in the FY of a few percent. In addition, the χ^2/ndf parameter for the sample without corrections would be larger, i.e., 1.46 instead of 1.21, showing that the compatibility of results improves when our corrections are applied. Alternatively, if the measurements of Kakimoto et al., Nagano et al. and Lefeuvre et al. are excluded to avoid any correction, an average of 7.00 ph/MeV is obtained with an associated uncertainty of 3.8% and $\chi^2/\mathrm{ndf} = 1.42$, which is fully compatible with the average for the whole sample with corrections.

Description	$\langle Y \rangle$ (ph/MeV)	Error	χ^2/ndf	Effect on $\langle Y \rangle$
Energy dependence	6.97	2.6%	0.97	-1.2%
Corr. to FLASH and AirLight	7.09	2.8%	1.21	+0.5%
Enlarged errors	7.04	2.9%	1.12	-0.3%
Correlations in E_{dep}	6.92 7.19	-	-	±1.9%

Table 1: Tests performed to evaluate the impact of possible factors on the average (see text for details). The average and associated uncertainty calculated from (1) and (3), respectively, as well as the χ^2/ndf parameter (4) in each test are listed in columns 2 to 4. The percentage difference in the average with respect to the result of figure 2 is shown in the last column.

We note that our average value and the measurement of Airfly are almost identical. This is partly due to the very low uncertainty of the measurement of Airfly, which dominates the average. Nevertheless, even though this measurement were excluded from the data sample, a very close value of 7.04 ph/MeV would be obtained with an associated uncertainty as low as 3.9% and $\chi^2/\mathrm{ndf} = 1.41$. Therefore, we can conclude that the result of Airfly is in full agreement with previous measurements, and the average should provide a better precision indeed.

On the other hand, the uncertainty resulting from equation (3) does not take into account some aspects. Four further tests have been performed with the purpose of obtaining a more realistic estimation of the uncertainty. These tests are summarized in table 1 and commented below:

1. As pointed out above, our MC algorithm predicts a weak energy dependence of the FY. If measurements are normalized to an electron energy of 100 MeV according to our simulation, the resulting average is 6.97 ph/MeV with an uncertainty of 2.6%. In addition, the compatibility of the data sample improves, i.e., χ^2/ndf is lowered from 1.21 to 0.97. This effect on the average and the associated uncertainty is mainly due to the normalization of the low-energy measurement of Dandl et al., which is somewhat larger than the remaining measurements at higher energies, in accordance with our theoretical predictions (figure 1).

2. Results on energy deposition obtained by FLASH [5], using EGS4, and by AirLight [6], using GEANT4, are 5% larger and smaller than ours, respectively (see [12, 14] for details). If the FY values reported by these experiments are modified according to our calculations on energy deposition, the resulting average is 7.09 ph/MeV, the uncertainty is 2.8% and the χ^2/ndf parameter is 1.21. That is, the average slightly increases by 0.5%, while there is no effect on the χ^2/ndf parameter.

3. Most authors neglected any error contribution due to the evaluation of the energy deposition, for which we conservatively estimate an uncertainty of 2%. In addition, differences in the fraction of nitrogen in atmospheric air and the synthetic air used in these measurements are usually ignored, whereas we have evaluated their effect on the FY to be as large as ±2% in some cases. If the quoted

error bars are enlarged by adding these two uncertainties in quadrature (when needed), an average of 7.04 ph/MeV, an uncertainty of 2.9% and a χ^2/ndf parameter of 1.12 are obtained. This increase in the experimental error bars has almost no effect on the resulting uncertainty of the average, because the use of equation (3) already accounts to a large extent for the possible underestimation of the experimental uncertainties (note that the χ^2/ndf is reduced). The decrease in the average is mainly due to the fact of giving a lower weight to the measurement of Lefeuvre et al., of which quoted uncertainty may be significantly underestimated.

4. The uncertainties in the evaluation of the energy deposition for the various experiments are expected to show some degree of correlation. In the most pessimistic hypothesis of fully correlation, all the measurements except for that of Dandl et al., which is not affected by this uncertainty, could be biased by up to $\pm 2\%$ in the same direction. This would result in a maximum deviation of $\pm 1.9\%$ in the average. Note, however, that different (fairly independent) MC codes are used to calculate the energy deposition. Moreover, even though the same MC code were used for all experiments, systematic errors in the calculated energy deposition would depend on their particular conditions (e.g., energy and geometry).

From these tests, it can be concluded that a critical analysis of the available measurements may lead to deviations of up to $\pm 2\%$ in the average. Therefore, we conservatively add an error contribution of $\pm 2\%$ in quadrature to the uncertainty calculated from (3). This results in an uncertainty of 3.5%, and therefore, our final average value of the FY for the 337 nm band in dry air at 800 hPa and 293 K is 7.06 ± 0.25 ph/MeV. The corresponding value at 1013 hPa and 293 K is 5.59 ± 0.20 ph/MeV.

Finally, it is worth noting that our theoretical yield value is in agreement with experimental results within uncertainties, which is a further support of the available experimental FY data. Details of these theoretical calculations and the estimation of uncertainties will be given in a paper in preparation.

6 Conclusions

In the present work, we present a reliable absolute FY value resulting from the weighted average of the available measurements of this parameter [1, 2, 3, 4, 5, 6, 7, 8]. Many possible error sources in these experiments have been investigated for the determination of a realistic uncertainty in the average. We also discuss the evaluation of the energy deposition and make use of simulation results from a detailed Monte Carlo algorithm [10, 12] that we have recently upgraded [14]. Corrections to some measurements [1, 2, 3] have been applied to eliminate a bias in the average.

Our average value of the absolute FY for the 337 nm band in dry air at 800 hPa and 293 K is 7.06 ± 0.25 photons/MeV. This result is fully compatible with that of Airfly (7.07 ± 0.29 ph/MeV), which is the most precise measurement so far and it has been adopted by Auger for the update of the energy scale [9].

Discrepancies in the energy scales of HiRes, TA and Auger might be partly due to the assumed FY in the shower reconstruction, and therefore, there is a growing interest in defining a common set of FY data (i.e., absolute value, relative spectrum and atmospheric dependencies) to be used by all cosmic-ray experiments in the near future [17]. This way it would, in principle, be possible to disentangle the contribution of the FY to the energy scale from other possible sources of discrepancy (e.g., optical calibration and reconstruction algorithm). Another strategy to determine the effect of the assumed FY data in the relative energy scale has been described in [18].

Finally, we would like to point out that this work aims at recognizing the work carried out by all the groups contributing to this field. Furthermore, the consistency of the available measurements allows us to assert a high level of confidence in our average and to quote an uncertainty as low as 3.5%.

Acknowledgment: This work has been supported by MINECO (FPA2009-07772, FPA2012-39489-C04-02) and CONSOLIDER CPAN CSD2007-42. We thank our colleagues of the Auger Collaboration for fruitful discussions and comments on this work.

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