SELF-CONSISTENT COSMOLOGY: AN INFLATIONARY ALTERNATIVE TO THE MINKOWSKIAN QUANTUM VACUUM

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ABSTRACT

A self-consistente cosmological scheme is proposed wherein the large-scale space-time curvature as well as the massive constituents are created simultaneously from an initially empty flat quantum Minkowski vacuum. The creation of each froces the other due to their gravitational coupling. Minkowski space is unable to sustain vacuum matter-gravitational interactions, provided the dimensionless parameter Km^2 exceeds the threshold value $Km^2 = 288\ r^2$. This parameter then plays the role of the critical point associated with a phase transition between Minkowski space and the self-consistently generated inflationary De-Sitter space-time.

I intend to describe in these notes the considerations we have recently made concerning what I shall call: the cosmological History of the Universe. By the latter, I mean that we are concerned with the global, large-scale structure of

Space-Time, its evolution and above all its origin, and not with the local small scale gravitational phenomenons. Our main concern is the following: why is there something rather then nothing, or, in other words, why is there a universe (at least one !) rather then the vacuum; moreover what is the mechanism responsible for the emergence of this "something" from "nothing", thereby giving later on rise to the presently observed constituents of our surrounding universe; last but not least, what is the nature of this primordial "nothing" ?

With this end in view, I intend to show you as a first step that the general cosmological problem is reducible to a quantum dynamical problem unfolding in the usual flat Minkowskian field theory. For that, I shall first of all recall very quickly the main properties characterizing the universe we observe today, on the large-scale here considered. First of all, we learned from the Hubble experimental discovery of galaxies recession that the universe is evolutive and not static (as was believed by the physics community up to Hubble's time) and, more precisely, that it is presently in an adiabatic expansion stage. The latter is characterized by the well-known Hubble's law, which asserts that the relative escape velocity of two galaxies is proportional to their mutual separation, the proportionality time-dependent coefficient being designated as the Hubble function H(t). Moreover, it was recognized after a major discovery by Pezias and Wilson that our universe is filled up with an isotropic homogeneous black-body radiation, the 2.7° K cosmological photon gas. In addition to these two major discoveries, the increasingly sophisticated experimental observation points with great confidence towards an image of the universe which appears the black-body photons but also the distribution of all kinds of cosmological constituants. This situation is in fact characterized by a numerical value: that of the mean observed mass-energy density in the universe which is approximatively 10^{-30} gr/cm^3. I quote this number explicitly because it has a double significant implication: firstly, it allows the explicit determination of the ratio of the density of black-body photons to the corresponding density of (observed) material constituents; it then appears that for every material particle in the universe, there are roughly 10^8 black-body photons, which implies that our universe is mainly populated with these photons rather than material particles. Moreover this number – the specific entropy of the universe – is in the present stage of cosmological adiabatic expansion, a constant in time and represents therefore one of the main fossils of the primevall configurations of the universe. Every plausible theoretical candidate to a cosmological history of the universe has to deliver this number. Another implication of the above-mentioned observed mean mass-energy density is the following: it fixes the global geometrical structure of the background; the latter is spatially open, flat or closed according to whether this density is lower, equal or greater than a given present critical energy density value. And here occurs the well known intriguing fact, namely that the experimentally observed density is very close to the critical one, so close that an increasing number of physicists are convinced today that rather than due to an unexpected accidental coincidence which would require the fine-tuning of some initial cosmological parameter, this property has a profound significance; for instance it expresses that both observed and critical densities are in fact strictly identical, implying thereby the spatial flatness of the large-scale cosmological background.

In short, all of these facts lead us presently to an image of our universe which is homogeneous, isotropic, open or more probably spatially flat and permanently expanding. I recalled to you briefly all this because the following fundamental property follows strightforwardly from these features: the global space--time geometry is conformally flat (conformally Minkowskian), namely its geometry differs from that of flat Minkowski space by only one degree of freedom; we denote it hereafter by a space-time depending scalar function so that:

$$ds^2 = e^{\lambda(x)} d\overset{o}{s}{}^2 \tag{1}$$

superscripts o will always refer to Minkowskian quanties. Equivalently, the components of the metric tensor takes the following form:

$$g_{\mu\nu} = e^{\lambda(x)} \overset{o}{g}_{\mu\nu} \quad . \tag{2}$$

It will appear in what follows that a very usefull parametrisation for the description of this conformal degree of freedom appears to be:

$$e^{\lambda(x)} = K/6 \ \phi^2(x) \tag{3}$$

where K is the gravitational coupling constant appearing in Einstein's equations and $\phi(x)$ is a massless real scalar field: the cosmological field.

Let us now firstly turn to the traditional dynamical description of a cosmological system whose material content will be described here in the simplest possible way, that is by a massive real scalar field $\psi(x)$. The General-Relativistic act giving rise to the usual Einstein equations controlling the dynamics of the gravitational-matter coupled system is:

$$S = \frac{1}{2} \int d^4x \quad \sqrt{-g} \, \{-K^{-1} R(g) + g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - m^2\psi^2 + \frac{R}{6} \psi^2\} \tag{4}$$

The first term in this action represents the free gravitational part and its typical negative sign reflects the universality of the attractive character of the gravitational interaction. The last term is the non-minimal coupling of the matter field to gravitation which guarantees the local scale-invariance in the case of vanishing mass; in the presence of this non-minimal coupling, the energy-momentum tensor resulting from the action (4) takes the following form:

$$T_{\mu\nu} = \partial_\mu\psi\partial_\nu\psi - \frac{1}{2} g_{\mu\nu}\psi^\sigma\psi_\sigma + \frac{1}{2} g_{\mu\nu}m^2\psi^2 + \frac{1}{6} [g_{\mu\nu} \Box - \partial_{\mu;\nu}]\psi^2$$

$$+ \frac{1}{6} (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R)\psi^2 \tag{5}$$

At this stage comes an essential point: if we want to extract explicitly the dynamical behaviour of the cosmological degree of freedom in a conformally Minkowskian background, then it is particularly usefull and instructive to perform it thanks to the following rescaling of the matter field:

$$\psi(x) = \psi(x) \sqrt{R/6} \quad \phi(x) \tag{6}$$

Indeed, the action (4) when rewritten in terms of this rescaled

$\psi(x)$-field together with the conformal metric (2) and (3), takes the following form:

$$S = \frac{1}{2}\int d^4x \sqrt{-\mathring{g}}\ \{[\mathring{g}_{\mu\nu}\psi_\mu\psi_\nu - \frac{R}{6}\ m^2\phi^2\psi^2 + \frac{\mathring{R}}{6}\psi^2] - [\mathring{g}^{\mu\nu}\phi_\mu\phi_\nu + \frac{\mathring{R}}{6}\phi^2]\ \} \tag{7}$$

where ψ_μ stands for $\partial_\mu\psi$.

The resulting action (7) hence describes the dynamics, unfolding in an underlying Minkowskian background, of two coupled scalar fields: the massive ψ-field and the massless cosmological field $\phi(x)$. The free action term of the latter appears with an overll negative sign which is reminiscent of the universality of the attractive character of gravitation. This sign is absolutely essential to the subsequent developments which I shall present in the rest of these lectures. Before discussing its consequences, let me remark the following feature: the particular form of the action (7) strongly suggests the following interpretation: it may be viewed as the phenomenological action describing a spontaneously broken local scale symmetry (Weyl) invariance in the framework of flat Minkowski space; the associated Nambu-Goldstone boson – the dilation – is in this case nothing but the cosmological field $\phi(x)$. This interpretation is only plausible if the non-minimal coupling term $\frac{R}{6}\psi^2$ is present in the action. It follows from this remark that if I had not any interest in the very beginning in a cosmological problem, but rather in flat space-time spontaneously broken Weyl invariance mechanism, then I would have written the action (7) in the first instance, as the phenomenological action describing this phenomenon. In that case, the inverse transformation to (6) together with the expression (2), which lead obviously to the General-Relativistic action (4) would play the role of the Higgs mechanism which, in this case, absorbs the Goldstone boson in the flat Minkowskian metric – thereby inducing a curved space-time. Is this description only a mathematical interpretation or is it the signal of some physical property ? I shall clarify this point later on. For the present, let me return to the action (7) as describing the cosmological dynamics in the Minkowskian background. The latter property implies that the usual concepts of causality and conservation laws, as formulated in the usual flat space-time language, are directly applicable; but what is the price we have to pay for these advantages: the mass term couples to the cosmological field $\phi(x)$ playing thereby the role of a time--depending interaction lagrangian. It is precisely this term which will be, as will be shown below in the case where the matter ψ-field will be treated quantum-mechanically, responsible for the non-conservation of the number of the massive quanta, hence particle creation. This mechanism is intimately linked to the above-mentioned overall negative sign accompanying the free cosmological-field action; its dynamical significance is indeed that the ϕ-field carries an opposite energy, hence negative energy, with respect to any conventional scalar field and in particular to the ψ-field. As a direct consequence, this unconventional sign opens the way to a possible spontaneous emergence of massive matter in an initially empty Minkowski space: its positive energy being exactly and permanently compensated by the negative energy associated to the cosmological

ϕ-field, hence by the global large-scale structure of space-time. Indeed, as I mentioned before, the conservation laws associated to action (7) are expressed in usual Minkowskian form, namely:

$$\partial_\mu \, [T^{\mu \text{ total}}_\nu] = \partial_\mu \, [T^{\mu \text{ (matter)}}_\nu + T^{\mu(\phi)}_\nu] = 0 \quad . \qquad (8)$$

Hence, from the restricted point of view of the energy-momentum conservation laws, nothing forbids a priori a non-trivial realisation of these laws; in such a case the total T^{μ}_{ν} (total) would permanently keeps its Minkowskian zero energy-momentum value. In spite of this global energy-momentum degeneracy of the matter-gravitational system with respect to empty Minkowski space, the two interacting parts $T_{\mu\nu}{}^{\text{(matter)}}$ and $T_{\mu\nu}{}^{(\phi)}$ will acquire non zero values starting at a given time, call it to. Hence, before the time to, the system is strictly Minkowskian with both parts of its total energy-momentum vanishing separately, whereas after time to the two matter and gravitational contributions $T^{\mu \text{ (matter)}}_\nu$ and $T^{\mu(\phi)}_\nu$ respectively are separately non vanishing although their sum is still zero. Explicitly we then have:

$$\begin{cases} t \le t_0 & T^{\mu\text{(matter)}}_\nu + T^{\mu(\phi)}_\nu = 0 \; . \\ t > t_0 & T^{\mu\text{(matter)}}_\nu \neq 0 \;\; , \;\; T^{\mu(\phi)}_\nu \neq 0 \;\; \text{but: } T^{\mu(m)}_\nu + T^{\mu(\phi)}_\nu = 0 \end{cases} \qquad (9)$$

The transition between these two regimes unfolds continuously at time $t = t_0$. This possible realisation of the conservation laws (9) would then correspond to a simultaneous emergence of massive matter constituants together with curved space-time background, out of an empty flat Minkowski space-time vacuum. Moreover, this process occurs without violation of energy-momentum conservation, its total net amount keeping its initial Minkowskian zero value. In other words, the positive energy carried by the emerging massive matter constituants is exactly compensated by the negative energy associated to the corresponding induced large-scale cosmological curvature. The latter appears in this way as an available internal intrinsic degree of freedom of the matter--gravitational system which allows the appearance ex-nihilis of massive matter. More precisely, it represents a negative energy reservoir from which the system is able to extract positive energy in order to create real massive particles. In other words, if the matter ψ-field is treated quantum-mechanically, there is no external source required to convert the virtual pairs populating its vacuum into real ones, but the internal global curvature degree of freedom itself. I will denote from now one such a simultaneous creation of matter and curvature as a self-consistent cosmological mechanism.

Taking all of that into account, the central problem which is to be considered is the following: is the self-consistent scheme, although compatible with the conservation laws, possibly realised dynamically, or in other words: do the equations of motion of the two coupled fields ψ and ϕ possess realisations, solutions corresponding to such a mechanism ? In order to obtain an answer to this essential question, let me first deduce from the stationarity of action (7) with respect to ψ and ϕ respectively, their equations of motion. From now on, I treat this problem explicitly in the context of semi-classical gravity, wherein the matter field ψ is treated quantum-mechanically, whereas the gravitational part, here represented by the massless ϕ-field, is treated classically. The equations of motion are:

$$\Box \psi + \frac{Km^2}{6} \phi^2 \psi = 0$$

$$\Box \phi - \frac{Km^2}{6} \phi <\psi^2>^s = 0 \qquad (10)$$

Let us first analyse the structure of these equations as well as the significance of the various symbols therein: I write the symbol "mean value" < > for ψ and not ϕ because, as explained previously, we are in the context of semi-classical gravity. The massive ψ-field being a quantum field, we shall be, as usual, confronted with the appearance of divergencies in the explicit evaluation of mean values like for example $<\psi^2>$; hence these equations (10) will acquire a physical meaning only after a well-defined subtraction procedure will be designed; the resulting subtracted corresponding value will be denoted by $<\psi^2>^s$. It is well known that it is always a very delicate problem to define such a procedure in an ambiguity-free manner and this is particularly the case in curved background. But in the present case, and this constitutes a very important property, there exists a physical guide which is intrinsically linked to the self-consistent idea. It works in the following way: it follows straightforwardly from the action (7) that in the case where $\phi(x)$ = const. = $\sqrt{6/K}$, then this action acquires the traditional form describing a free massive ψ-field in Minkowski space. Hence, under these circumstances, if the quantum state of the system is at a given time the vacuum, it will remain so as time elapses because there are no external sources which are able to provide energy-momentum required to create real massive pairs out of this vacuum. Moreover, the dynamical equations (10) reduce then to an equation for the free ψ-field alone, with an additional constraint which is $\langle\psi^2\rangle^s = 0$. Moreover ϕ = const. implies Λ = const., so that by equation (1), ds^2 reduces to $d\dot{s}^2$, hence Minkowskian geometrical background. It follows from all these facts that Minkowski space-time quantum vacuum, namely the geometrical Minkowski background in the presence of the quantum vacuum state for the quantum massive ψ-field, is a dynamical realisation – let me call it: the trivial solution – of the equations of motion (10), provided $\langle\psi^2\rangle^s = 0$. It follows from this that the general prescription for the subtraction procedure defining $\langle\psi^2\rangle^s$ must be such that if ϕ = const., then $\langle\psi^2\rangle^s$ vanishes. I will show you further on that this condition defines a privileged physical subtraction procedure in all non--trivial cases, that is cases corresponding to $\phi(x) \neq$ const. But, and this is of course the essential point, are there non trivial cases ? Namely are there non trivial (non-Minkowskian) solutions to the equations (10) ? In other words, these questions lead us to the following one: are there possible quantum dynamical behaviours of the quantum ψ-field coupled to the calssical ϕ-field which are able to sustain self-consistently non vanishing values for $\langle\psi^2\rangle^s$? If yes, are some of them continuously linked to the Minkowskian initial conditions, and last but not least: is there some stability argument which favours non trivial dynamical realizations above the trivial Minkowski vacuum solution ? All of these questions are linked to the detailed mechanism by which virtual particles populating the Minkowskian quantum vacuum are possibly converted to real ones at the expanse of curving space-time. Therefore, this problem is strongly dependent on the quantum bahaviour of the ψ-field in the presence of a space-time varying cosmological $\phi(x)$-field. Our main task is consequently to analyse in detail the quantization of the ψ-field in these circumstances. Once more, the Minkowskian formulation will be here of crucial help in that it will allow us to proceed along the conventional pattern of quantum field theory (in the presence of time--dependent interaction) as formulated in flat space-time.

THE QUANTIZATION PROCEDURE

I shall restrict the explicit quantization procedure to spatially flat conformally spaces. I insist on the fact that this represents only a technical simplification in the presentation of the results which are mainly unchanged in the case of an open universe. Moreover, in view of what have been said in the beginning of these lectures, the spatially flat structure plays probably a privileged role in cosmology so that this represent possibly not a restriction at all. Mathematically this implies that the ϕ-field or the λ function are only time t-dependent. I recall that the equation of motion for the matter ψ-field is

$$\Box \psi + m^2 e^{\lambda(t)} \psi = 0 \quad . \tag{11}$$

We then define a basis of functions $\{\psi_{\underline{k}}(\overline{x};t)\} = \{\psi_{\underline{k}}(\underline{x})\zeta_{\underline{k}}(t)\}$ with the help of the usual eigenfunctions $\psi_{\underline{k}}(\underline{x}) = e^{i\underline{kx}}/(2\pi)^{3/2}$ of the ordinary laplacian:

$$\Delta\phi_{\underline{k}}(\underline{x}) = - k^2 \phi_{\underline{k}}(\underline{x}) \quad . \tag{12}$$

This then fixes the time-dependence of the function $\dot{\zeta}_{\underline{k}}(t)$ to be controlled by the following equation:

$$\ddot{\zeta}_{\underline{k}} + \omega_{\underline{k}}^2(t)\zeta_{\underline{k}} = 0 \quad . \tag{13}$$

Where

$$\omega_{\underline{k}}^2(t) = k^2 + m^2 e^{\lambda(t)}$$

and a dot $\cdot$ means d/dt. It is directly obvious at this level that the dynamical system (in momentum space) is equivalent to a collection of free harmonic oscillators whose natural frequencies are time-dependent, thereby inducing a population of the modes, or equivalently a creation of massive particles. I shall precisely analyse this behaviour in some detail and will proceed in the easiest way (in this case), namely the canonical formalism. The natural canonical conjugate variables appear to be $q \equiv \psi$ and $p \equiv \dot{\psi}$. The construction of the associated Hamiltonian requires the mixed T_0^0 component of the energy--momentum deduced from the action (7). After some computations, the Hamiltonian takes on the following form:

$$H = \int d^3x \; T_0^0 = \frac{1}{2} \int d^3x \; [\dot{\psi}^2 + m^2 e^{\lambda}\psi^2 - \psi\Delta\psi] \quad . \tag{14}$$

It is an elementary exercice to check that $H \equiv p\dot{q} - L$ and that the canonical Hamiltonian equations $\partial H/\partial p = \dot{q}$ and $\partial H/\partial q = -\dot{p}$ are fulfilled (modulo the equations of motion).

Up to this point, we managed a classical Hamiltonian formalism. Nothing particular referred to the quantum nature of the ψ-field. It is precisely at this stage that the matter field quantum structure is imposed by requiring the equal-time commutation relations:

$$[\psi(\underline{x};t);\dot{\psi}(\underline{x}';t)] = i\delta^{(3)}(\underline{x}-\underline{x}') \quad . \tag{15}$$

This automatically promotes the previously obtained Hamiltonian (14) to the status of the time-displacement operator; it is indeed straightforward to check that the relation (15) implies that:

$$i[H;\psi(\underline{x};t)] = \dot{\psi}(x;t) \tag{16}$$

We now use, as usually, the basis $\{\psi_k(\underline{x};t)\}$ to define the creation and annihilation operators for the quanta of the ψ-field as follows:

$$\psi(\underline{x};t) = \int d\underline{k}\ \{\phi_k(\underline{x})\zeta_k^*(t)a_k + c.c.\} \quad . \tag{17}$$

The requirement of the usual commutation relation $[a_{\underline{k}};a^+_{\underline{k}'}] = \delta_{\underline{k}\underline{k}'}$ leads then, when combined with the commutation relations (15) to the normalisation of the ζ_k functions:

$$W = \zeta_k^*\dot{\zeta}_k - \zeta_k\dot{\zeta}_k^* = i \tag{18}$$

where W is the corresponding Wronskian, conserved by virtue of the equations of motion.

Our main taks is now to express the Hamiltonian in terms of creation and annihilation operators. This is deduced from (14) and (17) after a long but simple computation and delivers finally the following expression:

$$H = \int d\underline{k}\ \omega_k\ [(N_k + \tfrac{1}{2})E_k(t) + F_k(t)a_k^+ a_{-k}^+ + \text{c.c.}] \tag{19}$$

where:

$$\begin{cases} E_k(t) = \dfrac{|\dot{\zeta}_k|^2 + \omega_k^{\,2}(t)|\zeta_k|^2}{\omega_k} \\ F_k(t) = \dfrac{\dot{\zeta}_k^{\,2} + \omega_k^{\,2}\ \zeta_k^{\,2}}{\omega_k} \end{cases} \tag{20}$$

The particle number operator N_k is defined, as usual, by

$$N_k = a_k^+\ a_k \quad .$$

In the particular case of a static space-time, namely when A and ω are time independent, the ζ_k-functions reduce by equation (13) to ordinary plane-waves:

$$\zeta_k = e^{i\omega_k t} \Big/ \sqrt{2\omega_k} \tag{21}$$

But this implies in turn from relations (20) that:

$$\begin{cases} E_k(t) = 1 \\ F_k(t) = 0 \end{cases} \tag{22}$$

Hence, in the static case, the Hamiltonian (19) reduces to

$$H = \int d\underline{k}\ \omega_{\underline{k}}\ (N_{\underline{k}} + \tfrac{1}{2}) \quad . \tag{23}$$

In an expanding universe on the contrary, E is time-dependent and $F_k(t)$ does not vanish. It follows from these remarks that the

Hamiltonian commutes with the particle-number operator in the static case whereas it does not in the case of an expanding universe. A direct corollary of this property is that the number of ψ-particles, whereas conserved in the static case (for example in the case of empty Minkowski space), is not conserved in the case of an expanding universe. Hence, it is the space-time expansion which drives the massive particle production mechanism. Let me insist here on the following fact: had we included from the very baginning massless fields into the dynamics described by the action (4), they would not be affected by the rescaling transformation (6) – i.e. they would not couple to the cosmological field – because of their local scale (Weyl) invariance; hence, only massive particles are produced by the cosmological expansion. There is nevertheless one exception: the gravitons themselves which are produced together with massive particles because of the absence of Weyl invariance for the gravitational action (the gravitational coupling constant is a dimensional one).

In order to exhibit explicitly the rate at which the massive ψ-particles are produced for a given expansion we need to diagonalize the Hamiltonian (19); we proceed traditionally by performing an adequate Bogoliubov transformation on the (a,a^+)-operators; this has the form:

$$a^+_k = \alpha_k(t)\beta^+_k + \;_k(t)\beta_{-k} \quad . \qquad (24)$$

It is then required that the β-operators associated to the proper modes of the system be chosen in such a way as to cancel the non-diagonal terms, i.e., the functions $F_k(t)$, as well as to insure that the functions $E_k(t) = 1$ in the static case, i.e.

W_k = cst. These two conditions together with the commutation relations

$$[\beta_k;\beta_k^+] = \delta_{kk'},$$

lead to the explicit determination of the transformation (24); the result is

$$\begin{cases} |\alpha_k(t)|^2 = \dfrac{E_k(t)+1}{2} \\ |\gamma_k(t)|^2 = \dfrac{E_k(t)-1}{2} \end{cases} \tag{25}$$

In terms of these β-operators, the Hamiltonian (19) takes then the following form:

$$H = \int d\underline{k}\ W_{\underline{k}}(t)[N_{\underline{k}}(t) + \tfrac{1}{2}] \tag{26}$$

where the number operator $N_{\underline{k}}(t)$ associated to the proper modes $\beta_{\underline{k}}$-operators is given by $\beta_{\underline{k}}^+\ \beta_{\underline{k}}$. Obviously, owing to our generic Minkowskian reduction of the cosmological problem, the quantization procedure simplifies greatly with respect to the usual curved space quantization. But moreover, and this is an essential point, this Minkowskian formulation allows a well-defined definition of the vacuum state of the system on the one hand, and a simple physical prescription for the removal of divergencies on the other. I will now successively clarify these two facts.

I recall here that our main concern was to explore the possibility of a possible transition at a given arbitrary time to between the Minkowskian empty quantum flat regime and a non trivial (i.e. non empty and curved) expanding universe (see relations (9)). This implies that for times $t \leq t_0$, the cosmological function λ is a constant and hence the ζ_k-functions are the plane-waves (21); this in turn implies the values for E and F as given by the relation (22), so that the Bogoliubov's transformation (24) obviously obliges the β-operators to be identical to the Minkowskian a-operators for times $t \leq t_0$, namely:

$$\beta_k(t) \equiv a_k \quad , \qquad t \leq t_0 \quad . \tag{27}$$

With all that in view, we naturally define the vacuum state Ω of the cosmological system (in Heisenberg picture) as the vacuum corresponding to the a-operators, namely

$$a|\Omega\rangle = 0 \quad . \tag{28}$$

This choice guarantees that the density-number of particles in each mode k, n_k, vanishes for $t \leq t_0$:

$$n_k(t) = \langle\Omega|\beta_k^+\beta_k|\Omega\rangle \equiv \langle\Omega|a^+a|\Omega\rangle = 0 \quad , \qquad t \leq t_0 \quad . \tag{29}$$

The population of the modes for $t > t_0$ is accordingly given by:

$$n_k(t) = |\alpha_k(t)|^2\langle\Omega|a_k^+a_k|\Omega\rangle + |\gamma_k(t)|^2.\langle\Omega|a_ka_k^+|\Omega\rangle = |\gamma_k(t)|^2 \tag{30}$$

Using then the relations (25) and (20), it follows finally that:

$$n_k(t) = \left|\frac{1}{2\sqrt{\omega_k}}\left(\frac{d\zeta}{dt} - i\omega_k\zeta_k\right)\right|^2 \quad . \tag{31}$$

The choice of the quantum Heisenberg state of the system fixes then the corresponding initial conditions to which are constrained the functions $\zeta_k(t)$; it follows indeed from the expression (31) that:

$$\frac{d\zeta_k}{dt} - iW_k\zeta_k = 0 \quad \text{for} \quad t \le t_0 \quad . \qquad (32)$$

In other words,(W$_k$ being time-independent for $t \le t_0$) the functions $\zeta_k(t)$ reduce to the plane-waves (21) for $t \le t_0$.

Let us now turn to the divergence removal problem: apart from the eigenmodes frequencies $W_{\underline{k}}(t)$ time-dependence, the form of the Hamiltonian (26) is identical to that of flat space--time; in the latter case, i.e. for $t \le t_0$, the usual procedure is to remove the zero-point energy $\frac{1}{2}\int d\underline{k}\ W_{\underline{k}}$. A physically natural extension of this procedure for $t > t_0$ is then to adhere to the same prescription for $t > t_0$, namely to subtract at each time t, the instantaneous zero-point energy at that time: $\frac{1}{2}\int d\underline{k}\ W_{\underline{k}}(t)$. We then proceed in the same way for the mean square value of the matter fiel $\langle\psi^2\rangle$ which straightforwardly appears to be:

$$\langle\psi^2\rangle = \int_0^\infty \frac{k^2 dk}{2\Omega^2}\,|\zeta_k|^2 \quad . \qquad (33)$$

The previously mentioned requirements on the structure of $\langle\psi^2\rangle^s$ (see the discussion following the equation (10)) are fulfilled if we subtract, analogously to the Hamiltonian case, the instantaneous "zero point trace":

$$\frac{1}{2}\int_0^\infty \frac{k^2 dk}{2\Omega^2}\,\frac{1}{W_k(t)} \quad ,$$

thereby implying that:

$$<\psi^2>^s = \int_0^\infty \frac{k^2 dk}{2\pi^2} \left[|\zeta_k|^2 - \frac{1}{2W_k(t)} \right] \quad . \tag{34}$$

It can be shown in the context of dimensional regularisation that these two subtractions are simultaneously realised by the inclusion in the action of a single cosmological constant counter-term.

This closes the quantization procedure adapted to our cosmological scheme. There is of course still one equation left, namely the semi-classical equation relating the classical cosmological field $\phi(x)$ to the quantum matter field $\psi(x)$ (see equations (10)):

$$\phi \,\Box\, \phi - \frac{Km^2}{6} \phi^2 <\psi^2>^s = 0 \quad ;$$

but we know by equation (3) that: $\phi = \sqrt{6/K}\, e^{\lambda/2}$; hence:

$$\frac{3}{K} e^{\lambda} [\ddot{\lambda} + \frac{1}{2} \dot{\lambda}^2] = \frac{Km^2}{6} \frac{6}{K} e^{\lambda} <\psi^2>^s \quad ,$$

so that finally the semi-classical equation reduces to:

$$3[\ddot{\lambda} + \frac{1}{2} \dot{\lambda}^2] = Km^2 <\psi^2>^s \tag{35}$$

where a dot means d/dt.

If we multiply both sides by the factor $s^{-\lambda}$, this relation reveals its General-Relativistic significance in terms of the initial curved space-time quantities, namely:

$$R = - K\, T^{(s)} \quad . \tag{36}$$

The equation of motion for the cosmological field $\phi(x)$ reduces thus to the usual trace of Einstein's equations, its source being provided by the subtracted trace of the energy-momentum tensor associated to the quantized matter field $\psi(x)$; this expresses clearly the semiclassical aspect of our scheme. In short, putting together all the previously obtained relations, the whole self-consistent cosmological problem is controlled by the follosing set of equations:

$$\left.\begin{array}{lr}
3[\ddot{\lambda} + \frac{1}{2}\dot{\lambda}^2] = Km^2 <\psi^2>^s & \text{(a)} \\
<\psi^2>^s = \int_0^\infty \frac{k^2 dk}{2\pi^2} [|\zeta_k|^2 - \frac{1}{2W_k(t)}] & \text{(b)} \\
\ddot{\zeta}_k + W_k^2 \zeta_k = 0 & \text{(c)} \\
W_k^2 = k^2 + m^2 e^{\lambda(t)} & \text{(d)} \\
\dot{\zeta}_k \zeta_k^* - \dot{\zeta}_k^* \zeta_k = i & \text{(e)} \\
\dot{\zeta}_k - i W_k \zeta_k = 0 \quad , \quad t \le t_0 & \text{(f)}
\end{array}\right\} \quad (37)$$

In view of this set of equations (37), I am now able to formulate more precisely what I denote by a self-consistent cosmology: a given cosmological function $\lambda(t)$ will be called self-consistent if it corresponds to a quantum production rate (equations (37) (b) → (f)) induced by the corresponding cosmological expansion, whose feedback response regulated by the semi-classical Einstein's trace equation ((37)(a)) is just the one needed to sustain precisely this expansion.

We hence arrive at this point at a stage where the elf-consistent cosmological scheme which was previously shown to be consistent with the restricted conservation laws is explicitly formulated dynamically; consequently, our central consideration now concerns the possible existence of non-trivial (i.e. non Minkowskian) solutions to the equations ((37)(a) + (f)). According to these equations, there is an a priori intringuing fact: the left-hand side of the equation ((37)(a)) is independent of the dimensionless (in natural units $\hbar = C = 1$) parameter Km^2 as well as on the mass parameter m while at the same time the right-hand side is explicitly dependent on Km^2 on the one hand, and implicitly dependent on m through the expression for $<\psi^2>^s$, on the other. This implies that the existence of any self-consistent cosmological function $\lambda(t)$ carries a compatibility condition in the form of an eigenvalue condition on Km^2. I shall exhibit this property explicitly in what follows. There is of course no general method for solving such a highly non linear set of equations, nor to find an existence theorem. We shall therefore adopt two complementary attitudes: firstly, I shall give explicitly an exact solution which satisfies all requirements of the set of equations (37); its physical interest and its possible uniqueness will be discussed later on. Secondly, I shall develop a perturbation scheme, which although restricted by its perturbative character, will appear to be unexpectedly powerful, not only for the solution-finding problem but also for the understanding of the profound significance of the self-consistent mechanism on the one hand, and of the subtraction procedure on the other.

AN EXACT SOLUTION: THE SPATIALLY-FLAT DE-SITTER SPACE TIME

The function $e^{\lambda(t)} = t^{*2}/t^2$ (38) which describes the Euclidean De-Sitter space time as expressed in conformal coordinates is an exact self-consistent solution. Indeed, the equation ((37)(c)) then takes the following form:

$$\ddot{\zeta}(t) + (k^2 + \frac{m^2 t^{*2}}{t^2})\, \zeta(t) = 0 \tag{38}$$

If we define y(t) by:

$$\zeta(t) = t^{1/2}\, y(t) \tag{39}$$

$$t^2 y + ty + (k^2 t^2 + m^2 t^{*2} + \tfrac{1}{4})y = 0 \tag{40}$$

Defining then:

$$z(t) = kt \qquad \text{and} \qquad V^2 = m^2 t^{*2} - \tfrac{1}{4} \;,$$

the equation (40) reduces to:

$$z^2 y'' + zy' + zy' + (z^2 + V^2)y = 0 \tag{41}$$

where ' denotes d/dz.

The general solution of the equation (40) is a linear combination of Hankel functions with imaginary indices $H^{(1)}_{iv}(z)$ and $H^{(2)}_{iv}(z)$, more precisely:

$$y(t) = a\, H^{(1)}_{iv}(z) + b\, H^{(2)}_{iv}(z) \quad , \tag{42}$$

where:

$$H^{(1)} \equiv e^{-\pi k/2}\, H^{(1)} \qquad , \qquad H^{(2)} \equiv e^{\pi k/2}\, H^{(2)} \tag{43}$$

These modified Hankel functions $H^{(1)}$ and $H^{(2)}$ are complex conjugates of each other:

$$(H^{(1)}_{i\nu}(z))^* = (H^{(2)}_{i\nu}(z)) \quad , \tag{44}$$

Hence,

$$\zeta(t) = \sqrt{t}\ (a\ H^{(1)}_{i\nu} + b\ H^{(2)}_{i\nu}) \quad . \tag{45}$$

The normalization as well as initial conditions ((37)(e);(f)) fixes then the linear combination (45) to be:

$$\zeta(t) = \sqrt{t}\ \ \sqrt{\pi/4}\ \ H^{(1)}_{i\nu}(z)\ e^{-\nu\pi/2} \quad . \tag{46}$$

It follows from this expression that:

$$<\psi^2>^s = \int_0^\infty \frac{k^2dk}{2\pi^2} \left[t\ H^{(1)}_{i\nu}(z)\ H^{(2)}_{i\nu}(z)\ \frac{\pi}{4} - \frac{1}{2\sqrt{k^2+m^2\ \ t^{*2}/t^2}} \right] \tag{47}$$

Using the definition $z = kt$, this expression possesses a remarkable t-scalling property, i.e.

$$<\psi^2>^s = \frac{1}{t^2} \left[\int_0^\infty \frac{z^2dz}{2\pi^2} \left(\frac{\pi}{4}\ H^{(1)}_{i\nu}(z)\ H^{(2)}_{i\nu}(z) - \frac{1}{2\sqrt{z^2 + \nu^2 + \frac{1}{4}}} \right) \right] \tag{48}$$

The integral in the equation (48) is not only finite but is manifestly also t-independent; according to (48), the trace equation ((37)(a)) hence takes the form:

$$\frac{K}{t^2} = Km^2\ \frac{1}{t^2}\ f\ (m^2t^{*2}) \quad . \tag{49}$$

Owing to the scale property exhibited in equations (47) and (48), the t-dependence finally cancels out, leading to

$$\frac{K}{Km^2} = f(m^2 t^{*2}) \qquad (50)$$

Self-consistency is thereby realized provided the functional relationship expressed by equation (50), between Km^2 and t^{*2} is fulfilled. The explicit determination of the integral $f(m^2 t^{*2})$ leads to the following result:

When t^* runs in the interval $-\infty \rightarrow 0$ (which corresponds to the physical domain for the conformal t-time in the De-Sitter solution (38), then f covers the (semi-open) interval: $(1/24 \rightarrow 0]$; this in turn implies that the existence condition for the De-Sitter cosmological solution takes the form of an eigenvalue condition on Km^2, namely:

$$Km^2 > 288\ \pi^2 \quad . \qquad (51)$$

The mass m of the self-consistently created particle must be greater than a given threshold mass m_{th} defined by:

$$Km_{th}^2 = 288\ \pi^2 \quad . \qquad (52)$$

This threshold mass corresponds to $1.3 10^{20}$ GeV or $\approx 3 10^{-4}$ gr; I shall return later on to a possible physical interpretation of these supermassive self-consistently created particles.

If we describe these particles phenomenologically as a perfect fluid characterized by a traditional energy-momentum tensor, then the corresponding energy density σ on the one hand and the pressure p on the other hand are respectively given by thoir quantum expressions:

$$\sigma = \int_0^\infty \frac{k^2 dk}{2^{\ 2}} n_k(t) W_k(t) \tag{53}$$

and

$$\tau = \sigma - 3p = m^2 <\psi^2>^s e^{-\lambda(t)} \tag{54}$$

which are, due to their self-consistent character, identical to the corresponding Einstein's General-Relativistic classical expressions:

$$K\sigma = \frac{3}{4} \dot{\Lambda}^2 e^{-\Lambda} \tag{55}$$

$$Kp = e^{-\Lambda}(- \ddot{\Lambda} - \dot{\Lambda}^2/4) \quad . \tag{56}$$

It follows then from (53)(54), or equivalently but more straightforwardly from (55) and (56) that the self-consistently created perfect fluid satisfies the De-Sitter equation of state:

$$\sigma + p = 0 \quad . \tag{57}$$

Moreover, the energy-density is given by:

$$K\sigma = \frac{3}{t^{*2}} = \text{const.} \tag{58}$$

The relations (57) and (58) have two important consequence: firstly, it follows from (58) that the limit $t^{*2} \to \infty$ corresponds to the "Minkowskian" limit $\sigma \to 0$; this then implies (see our previous discussion on the self-consistent existence condition $Km^2 > 288\pi^2$) that the threshold mass (52) corresponds to this Minkowskian vacuum limit $\sigma \to 0$. I shall discuss later the important implication of this property.

Secondly, the presdure p associated to the fluid of produced particles is negative! The negativity of the pressure reflects in fact the particle quantum particle creation mechanism; this is obvious by the conservation law, as expressed phenomenologically:

$$\frac{d}{dt} [\sigma\, e^{3\lambda/2}] = - p \frac{d}{dt} [e^{3\lambda/2}] \quad , \tag{59}$$

where $e^{3\lambda/2}$ is the Robertson-Walker function in these coordinates. Clearly, an expanding universe (i.e. a growing cosmological function $\lambda(t)$) which forces particle creation (i.e. a growing energy per comoving cell $\sigma\, e^{3\lambda/2}$) implies negative pressure p. Moreover the negativity of the pressure is sufficiently important (recall $\sigma + p = 0$) to violates the premises of the general positivity theorems by Hawking and Penrose; these theorems imply the inevitability of a big-bang occurring in a finite past. Therefore in our case, such a singularity is not required and is moreover not even present. Concerning this last point, let me insist on the following: the big-bang singularity corresponds to $\lambda = -\infty$ or equivalently to the vanishing of the cosmological field $\phi = 0$. And this precisely represents a singularity in the rescaling transformation (6) which then in turn forbids the whole dynamical Minkowskian generic reduction (the passage from the action (4) to the one given by equation (7)) and a fortiori the whole self-consistently generated universe concept.

To sum up, we arrive at a stage where we have shown that there exists at least one non-trivial realisation of the self-consistent mechanism; but this fact may possibly represent a progress in the initial question: "why is there something rather than nothing ? " Only if the cosmological system has some good physical reason to choose the non-trivial realisation rather than the Minkowskian trivial one; in other words, why does the system not persist to fluctuate quantum-mechanically in the quiete empty Minkowski empty space-time rather than to jump into a energetically degenerate non trivial, non empty curved space-time self-consistent solution. Why does the virtual massive pairs populating a quantum initial Minkowskian vacuum have a spontaneous tendency to be converted into real ones, extracting accordingly the required energy from the Minkowskian geometrical background thereby gradually curving it. This fundamental question finds its answer in the following completely unexpected property: empty quantum Minkowski space--time appears to be unstable in the presence of a quantum massive scalar field coupled (semi-classicaly) to gravitation, provided the dimensionless parameter Km^2 exceeds the threshold value $Km^2_{th} = 288\,\pi^2$!. Hence, as I shall now prove, it appears that this instability condition is identical with the existence condition (relations (51),(52)) for the De-Sitter self-consistent cosmology ! This is in strong analogy with a phase transition, with critical point at Km^2_{th}, the two phases being Minkowski and De-Sitter spaces. From this point of view, the self-consistent realization appears as a non-trivial alternative to the quantum vacuum. I shall now present a proof of the instability of

Minkowsky space subject to the condition:

$$Km^2 > Km_{th}^2 = 288\ \pi^2 \quad .$$

For this purpose we analyse the linearized dynamical behaviour of a small time-dependent (global) conformal perturbation $\delta(t)$ affecting the Minkowskian background for $t > t_0$, t_0 being an arbitrary time – say $t_0 = 0$:

$$\phi(t) = (6/K)^{1/2} + \delta(t) \quad ; \quad \delta(t) = 0 \quad , t \leq 0 \tag{60}$$

The dynamical equations (10) accordingly linearized become respectively:

$$\Box\, \psi + m^2\ [1+2(K/6)^{1/2}\ \delta(t)]\psi = 0 \tag{61}$$

$$\ddot{\delta} - m^2[(K/6)^{1/2} + K/6\ \delta(t)]\langle\psi^2\rangle^s = 0 \tag{62}$$

According to the relation (69), the imperturbed matter field ψ_0 (for $t < 0$) is the usual free-field solution (in Heisenberg representation), namely:

$$\psi = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{e^{i\underline{kx}}}{(2Wk)^{1/2}}\ [a^+_{\underline{k}}\ e^{i0t} + \text{c.c.}] \tag{63}$$

Equation (61) then fixes the corresponding response (for $t > 0$) of the matter field to the perturbation (60), $\delta\langle\psi^3\rangle^s$, where the subtraction, following our general prescription, eliminates the zero-point energy corresponding to the effective mass $\dot{m}^2(t) = m^2[1+2(K/6)^{1/2}\ \delta(t)]$. By requiring also the initial condition corresponding to (60), i.e. $\delta(0) = 0$, we finally obtain:

$$\delta\langle\psi^2\rangle^s = \frac{m^2}{4\pi^2} \left(\frac{K}{6}\right)^{1/2} \int_0^t dt_1 \, \dot{\delta}(t_1) \int_0^\infty \frac{k^2 dk}{W^3} \cos 2W(t-t_1) \quad . \tag{64}$$

This expression hence represents the subtracted response of $\langle\psi^2\rangle^s$ to the global conformal perturbation $\delta(t)$. The feedback response to $\delta\langle\psi^2\rangle^s$ as given by equation (64) for the geometrical perturbation $\delta(t)$ (itself) is then obtained from equation (62):

$$\ddot{\delta} = m^2 \left(\frac{K}{6}\right)^{1/2} \left[\frac{m^2 (\frac{K}{6})^{1/2}}{4\pi^2} \int_0^t dt_1 \, \dot{\delta}(t_1) \int_0^\infty \frac{k^2 dk}{W^3} \cos 2W(t-t_1)\right] + O(2) \tag{65}$$

After integrations by parts and taking once again into account the initial conditions, this finally leads to:

$$\ddot{\delta}(t) = \frac{m^4 \frac{K}{6}}{8\pi^2} \left[\dot{\delta}(0) \int_0^\infty \frac{k^2}{W^4} \sin 2Wt + \int_0^t dt_1 \, \ddot{\delta}(t_1) \int_0^\infty \frac{k^2}{W^4} \sin 2W(t-t_1)\right. \tag{66}$$

This integro-differential equation is of the convolution type for the Laplace transform so that it reduces to the simple algebraic relation:

$$\ddot{\delta}(s) = \lambda\dot{\delta}(0) \, g(s) + \lambda\dot{\delta}(s) g(s) \quad , \tag{67}$$

where: $g(s)$ is the Laplace transform of the function

$$g(t) = \int_0^\infty \frac{dk k^2}{W^4} \sin 2Wt \; ;$$

Hence:

$$g(s) = \int_0^\infty dt\, e^{-st}\, g(t) = \frac{2}{s^3}\,\{(4m^2+s^2)^{1/2}\ \ln s/2m + (s^2/mm^2 + 1)^{1/2} - s\},$$

and

$$\lambda = Km^4/48\pi^2$$

contrary to the first impression, g(s) is a finite even function of s.

The maximum of the function g(s) is $g(0) = 1/\sigma\, m^2$. It follows obviously from (67) that:

$$(s) = \frac{\lambda\ \dot{\delta}(0)\ g(s)}{1 - \lambda\, g(s)} \quad . \tag{68}$$

Let me insist here on the fact that all these forms are the result of a subtle combination of the subtraction procedure together with the initial Minkowskian conditions in the framework of the self-consistent dynamical equations here under consideration. A direct consequence of the form of g(s) is the following: if $\lambda g(0) = Km^2/288\ ^2 > 1$, then $\ddot{\delta}(s)$ has two real symmetric poles and its inverse Laplace transform $\ddot{\delta}(t)$ (as well as $\delta(t)$), accordingly grows exponentially with time. On the contrary, in the case $Km^2/288\ ^2 < 1$, the function $\ddot{\delta}(s)$ has a double imaginary pole and $\ddot{\delta}(t)$ (as well as $\delta(t)$) exhibits bounded oscillating behaviour. Thus, as announced, there is a threshold value of Km^2 ($Km^2_{th} = 288\pi^2$) which renders the trivial Minkowski solution unstable. It must be stressed here that this analysis based on the solution (68) presupposes that at the initial time $t = 0$ the perturbation is such that its first derivative (at least) is not zero.

Hence, as announced previously, the existence of the non-trivial solution and the instability of the trivial one are subject to a common quantitative condition. Concerning the latter, let us stress the fact that no other physical parameter enters into the determination of this threshold value $Km^2_{th} = 288\, r^2$. This, then, is a fundamental dimensionless constant, characteristic of Minkowski space, which marks the dividing line between stable and unstable vacuum fluctuations of matter (in the present case limited for simplicity, to a scalar field. For other models of matter, there will possibly be some other values of Km^2_{th}).

The very fact that a common critical parameter value ($Km^2{}_{th}$) is attached to the self-consistent De-Sitter solution as well as to the (in) stability behaviour of Minkowski space, strongly suggests that it must be hidden implicitly within the dynamical equations themselves. It is in an attempt to clarify this point that we proceeded to a suitably adapted perturbative approach to these equations. This perturbative procedure which I shall introduce now will also unexpectedly provide a reinterpretation (at least to a given order) of the corresponding unfolding of the cooperative dynamical behaviour; moreover, it will appear that the perturbative scheme delivers (to the given considered order), the correct values for all exactly known situations (for example: the instability threshold of Minkowski space, the existence condition for the self-consistent De-Sitter solution, the correct expression for the trace anomaly – see later); this provides us, despite the restrictive character of an analysis based on a perturbative scheme, with strong confidence as regards to its predictive power. Let me now introduce the main points characterizing the above-mentioned perturbative technique:

a very powerful change of variables for this problem appears to be given by:

$$\begin{aligned} p &= k/m \\ W_p(t) &= p^2 + e^{\Lambda(t)} \\ z(t) &= m|\xi|^2 \quad . \end{aligned} \tag{69}$$

The idea of this change of variables is to deal only with dimensionless quantities p, W_p and z; this then implies that in a Laurent-type development of z in terms of the mass-parameter m^2 which is of dimension $\underline{1}$ namely:

$$z = z_0 + \frac{z_1}{m^2} + \frac{z_2}{m^4} + \ldots + \sum_{n=0}^{\infty} \left(\frac{z}{m^2}\right)^n z_n \tag{70}$$

Tha various coefficients z_n are respectively tensors of a definite corresponding order.

The differential equation for z follows directly from equation (13):

$$\frac{1}{2} zz - \frac{1}{4} z^2 - \frac{m^2}{4} + m^2 W_p^2 z^2 = 0 \quad . \tag{71}$$

When the development (70) is inserted in equation (71), this then gives rise to:

$$\frac{1}{2} \sum_0^{\infty} \frac{1}{(m^2)^n} \sum_0^n z_{n-\ell} \ddot{z}_\ell - \frac{1}{4} \sum_0^{\infty} \frac{1}{(m^2)^n} \sum_0^n \dot{z}_{n-\ell} \dot{z}_\ell - \frac{m^2}{4} + m^2 W_p^2 \sum_0^{\infty} \frac{1}{(m^2)^n} \sum_0^n z_{n-\ell} z_\ell = 0 \tag{72}$$

Proceeding then order by order the relation (72) delivers then the set of all z_n's; for example:

$$\begin{cases} z_0 = \frac{1}{2Wp} \\ z_1 = \frac{(e^{\Lambda})^{\cdot\cdot}}{16W^5} - \frac{5}{64}\,\frac{[(e^{\Lambda})^{\cdot}]^2}{W7} \\ \cdots\cdots\cdots\cdots \end{cases} \tag{73}$$

Moreover the expression for $<\psi^2>^s$ is obtained from (34) and (69) as:

$$<\psi^2>^s = m^2 \int_0^\infty p^2\, dp\, (zp - \frac{1}{2Wp}) \tag{74}$$

The value of z, up to a given order, according to (70) and (73) is then inserted into (74) thereby delivering the relation controlling the self-consistent dynamics up to this order; for example, to the order $O(\frac{1}{m^2})$, this leads to:

$$(\ddot{\Lambda} + \frac{\dot{\Lambda}^2}{2})(1 - \frac{Km^2}{288\pi^2}) = \frac{K\,e^{-}}{2880\pi^2}\,(-\,\Lambda^{(4)} + \Lambda^2\ddot{\Lambda}) \tag{75}$$

which is nothing but (in curved-space language)

$$-R\,(1 - \frac{Km^2}{288\pi^2}) = \frac{K}{2880\pi^2}\,(\Box R - \frac{1}{3}R^2 + R_{\alpha\beta}R^{\alpha\beta}) \tag{76}$$

or equivalently

$$R = -\,K_{eff}\,(\Box R - \frac{1}{3}R^2 + R_{\alpha\beta}R^{\alpha\beta}) \tag{77}$$

where the effective gravitational coupling constant K_{eff} is defined by

$$K_{eff} = \frac{K}{2880\pi^2(1 - \frac{Km^2}{288\pi^2})} \quad . \qquad (78)$$

Apart from exhibiting explicitly the threshold value $Km_{th}^2 = 288\pi^2$ in the dynamical equation (75) itself, this result calls immediately for the following comments: K_{eff} being positive or negative according to the condition $Km^2 < 288\pi^2$ or $Km^2 > 288\pi^2$, the effective gravitational interaction is accordingly attractive or repulsive. This shows a new interpretation on the stability property of Minkowski space (or equivalently on the existence condition of the self-consistent cosmologies). It appears indeed that the instability of Minkowski space sets up as soon as the effective interaction among virtual vacuum fluctuations becomes repulsive (at least to the lower perturbation order). This effective anti-gravitational repulsive interaction among virtual particles (of mass $m > m_{th}$) populating the initial Minkowski space prevents subsequent reannihilation and provides them the required amount of positive energy to convert them into real ones; this energy is of course extracted from the geometrical background. This appears to be the fundamental mechanism responsible for the breakdown of Minkowskian quantum vacuum, unable to sustain such quantum fluctuations. This some phenomenon of effective gravitational repulsion allows us to understand why the resulting new configuration (i.e. De-Sitter space) is in a stage of fast exponential (as measured in proper time) expansion: the massive created entities are subject by the self-consistent dynamics to the condition $m > m_{th}$ and hence interacts (in the same way as the virtual Minkowskian quanta) gravitationally repulsively, thereby preventing a slowing down of the Hubble function (as in the presente adiabatic exponential stage).

It results moreover from (77), that the source of the scalar curvature is that combination of second-order invariants which is precisely the trace anomaly associated with the residual massless part of the scalar field. Thus, at that order of perturbation, the mass of the massive scalar quanta is completely absorbed in a rescaling of the gravitational coupling constant, which describes then an anti-gravitational repulsive interaction among massless particles.

It was previously pointed out that the action (7) as expressed in terms of the rescaled field (6), may possibly be viewed as the phenomenological action describing a spontaneously broken Weyl symmetry phenomenon occurring in Minkowski space. The corresponding Goldstone boson — the dilaton field — is then the cosmological field e^{Λ}; therefore, only massless fields would in fact contribute to the corresponding underlying fundamental action. The possible relevance of this property expressed in the equation (77) to this interpretation is not clear and represents one of the presently investigated questions.

Let us now apply this perturbative scheme to the self--consistent De-Sitter solution: in this case, $R_{\alpha\beta} = -K\sigma\, g_{\alpha\beta}$ so that the equation (76) gives rise to the relation:

$$\mu = - \frac{K^2}{3.2880\pi^2}\,\sigma \qquad (79)$$

where

$$\mu = 1 - \frac{Km^2}{288\pi^2} \quad .$$

We hence recover in the equation (79) the previously mentioned existence condition for the self-consistent De-Sitter solution: $Km^2 > 288\pi^2$ and $Km^2 \to 288\pi^2$ for $\sigma \to 0$.

The perturbative scheme presented above lends itself also particularly suitably for the stability analysis of the non-trivial De-Sitter solution. This represents of course an important problem: the unstable character of Minkowski space poses an essential question, namely: is the self-consistent De-Sitter cosmology stable with respect to these fluctuations which precisely cause the fate of the flat Minkowskian vacuum ?

The idea is to perform small global conformal perturbations around the exact solution $\Lambda_e(t)$, this is realized more conveniently on $\exp(\Lambda)$ rather than on Λ itself, so that one poses:

$$e^{\Lambda(t)} = e^{\Lambda_e(t)} + \delta(t) \tag{80}$$

one then analyses the behaviour of the perturbation $\delta(t)$ as given by equation (75).

First of all, I recover in this way the already known exact result for the trivial Minkowski space-time solution; in this case, $e^{\Lambda_e} = 1$, so that equation (75) reduces to:

$$\delta^{(4)} = - \frac{1 - \frac{Km^2}{288\pi^2}}{\frac{K}{2880\pi^2}} \delta^{(2)} \tag{81}$$

which obviously leads to the critical value $Km^2 = 288\pi^2$, which borders the two regimes – stable and unstable – for the conformal fluctuations in this case. I shall now proceed along the same pattern in analyzing the (in)stability of the De-Sitter solution.

I recall that in this case: $e^{\Lambda_e} = t^{*2}/t^2$; I define henceforth the corresponding perturbation $\delta(t)$ by

$$e^{\Lambda(t)} = \frac{t^{*2}}{t^2} + \delta(t) \quad . \tag{82}$$

The function

$$\Delta(t) = \delta(t)\, e^{-\Lambda_e} = t^2/t^{*2} \;\; \delta(t) \tag{83}$$

is easily shown to be controlled by the quartic equation:

$$t^4\Delta^{(4)} - 5t^2\ddot{\Delta} + 10t\dot{\Delta} - 4\Delta = 0 \tag{84}$$

The general solution of this equation is given by

$$\Delta(t) = a_1 t^{-1} + a_2 t^4 + a_3 t^{\mu} + a_4 t^{\nu} \tag{85}$$

where μ, ν are respectively $\frac{3 \pm \sqrt{5}}{2}$.

A crucial role in fixing the (in)stability of the investigated solution is played by the physical relative energy-density perturbation $\delta\sigma/\sigma$; it follows straightforwardly from (85) that:

$$\delta\sigma/\sigma = -t^{\rho}(1+\rho) \quad , \tag{86}$$

where ρ is any exponent appearing in the expression of $\Delta(t)$ as given by (85); only one of them is negative, namely $\rho = -1$ which "freezes" completely the corresponding fluctuation. The three other exponents being positive, we have a corresponding polynomial damping (as Minkowskian t-time elapses asymptotically to $t \to 0$) for the corresponding perturbation. In conclusion, the De-Sitter solution appears to be stable (against this type of perturbations), in accordance with a previously mentioned result.

Besides from its stability, another essential question concerns the uniqueness of the De-Sitter solution. And once more, the perturbation scheme offers us unexpectedly a strong indication in this questionning; it leads indeed to a quite remarkable property shared by the matter-gravitational system. It appears indeed that the whole dynamics as regulated both by the self--consistently generated trace and energy-density equations, is subject to a dissipative conservation law ! In short, considering $\sqrt{H(t)} \equiv f$ as the basic dynamical variable, where H is the Hubble function, the complete set of self-consistent equations limited to the given order of perturbation reduces to:

$$\frac{d}{d\tau}\left[2\nu \dot{f}^2 + \frac{\mu}{2} f^2 + \frac{\nu}{6} f^6\right] = -12\nu\, f^2 \dot{f}^2 \qquad (87)$$

where $\nu = K/2880\pi^2$, $\mu = 1 - Km^2/288\pi^2$ and $\dot{} \equiv d/d\tau$, where τ is the proper time. This is nothing but a dissipative conservation law with the dissipative term $-12\nu\, f^2\dot{f}^2 = -3\nu\dot{H}^2$. The fact that the Hubble function H appears as a privileged degree of freedom which characterizes the self-consistent matter-gravitational dynamics is not surprising because it is the only fundamental scale available with respect to which the initial fluctuations can be compared. The potential $V(f) = \frac{\mu}{2} f^2 + \frac{\nu}{6} f^6$ which characterizes the dynamics controlled by equation (87) hence appears as the "effective potential" for the self-consistent cosmological problem (at that order of the perturbative scheme). It clearly gives rise to a spontaneously broken symmetry behaviour when μ is negative, i.e. $Km^2 > 288\pi^2$; the corresponding threshold parameter hence is identical with the previously obtained critical point for the phase transition between Minkowskian vacuum and the self-consistently generated cosmologies. It is obvious that if $\mu > 0$, the minimum of the potential $V(f)$ is located at $f = 0$, so Minkowski space-time is stable; if $\mu < 0$, the system jumps to the new stationary point $f^4 = -\mu/\nu$ the "true self-consistent ground-state" which is the De-Sitter space. It is straightforward to show that the dissipative term prevents oscillations around this "true ground-state". Hence, if $\mu < 0$ (i.e. $Km^2 > 288\pi^2$), Minkowski space is unstable and the universe inevitably tends to a De-Sitter space characterized by an energy-density $K\sigma = 3H^2$, with $H^2 = \mu/\nu$ in complete agreement with the previously obtained results. Moreover, the very existence of this potential $V(f)$ strongly suggests that the De-Sitter solution is the unique dynamical realisation of the proposed self-consistent mechanism.

We are obviously not presently in a De-Sitter exponentially expanding universe and certainly not surrounded by permanently created supermassive constituants. Our aim was nevertheless to construct a whole cosmological History which also encompasses the present adiabatic expansion stage of a universe filled up with ordinary matter and radiation. With that in view it is essential to understand why the super-massive particle production stage stops. The situation is the following: the self-consistently produced supermassive De-Sitter particles are interpreted as those required by the grand unification scheme, or alternatively as primitive black holes – the primeval source of temperature – whatever they are, they have a finite decay or black hole evaporation time which precisely corresponds to the De-Sitter production duration stage. Indeed, the decay provides massive ordinary elementary particles as well as photons; this then implies a breakdown of the eigenvalue self--consistent existence condition (the masses of the presently observed constituants of the universe lying of course under m_{th}) which inhibates the aboundant self-consistent feedback production mechanism; on the other hand, the De-Sitter equation of state is no longer realized (appearance of positive pressure associated for example the photons resulting from the decay process); these two facts imply that the cosmological behaviour is no more self--consistent and therefore, the dynamical equations controlling this behaviour lead no longer to a rescaled anti-gravitational coupling constant K_{eff} (see relation (78)). This then explains why the De-Sitter regime stops and why there is a turn-over to a slowing-down expansion rate (i.e. the presently observed adiabatic expansion). This decay process hence gives rise to the same consequences as those resulting from the release of the energy associated with the null Higgs field in the framework of Guth's traditional inflationary conjecture: in both cases, the exponentially driving mechanism disappears and the inflationary expansion stops (no more negative pressure nor cosmic repulsion) and an enormous amount of energy and entropy are then released. Hence, our scheme represents an alternative to Guth's idea that the energy density of the false vacuum associated to the supercooled phase drives the "steady-state stretch". Instead, it is the instability of Minkowski quantum vacuum which forces the system, in our context, to undergo above Km_{th}^2 , a phase transition from an initial Minkowski space to the self-consistent De-Sitter universe.

In conclusion, let us summarize the chain of facts presented above in the shape of the following cosmological history Minkowski space is unable to sustain vacuum matter-gravitational interactions and therefore transits to a new phase, the De-Sitter universe. The latter, an essential ingredient of inflationary type universes appears thus as the natural primeval stage of physical space-time. After decay of the primeval constituants, there is a turn-over to the present cosmological free expansion configuration. The universe built up in this way, appears thus as a non-trivial energetically degenerate alternative to the quantum flat vacuum.

Clearly the present developments are too embryonic to permit any kind of judgement or preference. What really are our very massive De-Sitter constituents (more than ten times the Planck mass) ? If they are indeed black holes, can they be simulated by a quantum field and if not is Minkowski space nevertheless unstable with respect to such massive fluctuations. In spite of all these uncertainties, it is very satisfactory that our self-consistent scheme leads naturally to a primeval inflationary stage, since for the present it is the only mechanism which offers the possibility of success in confronting the problem of causality posed by the big-bang.

We are not at all claiming to have obtained The Cosmological History of the Universe. Our more humble purpose is only to show that it is possible to conceive and to construc explicitly, in the framework of the traditional laws of Nature, a cosmological history of the universe which avoids the big-bang and a fortiori its "unpleasant" consequences, and moreover provides a "natural" inflationary primordial scenario.

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