Abstract:

Final state $q\bar{q}$ interactions give origin to non zero values of the off-diagonal element $\rho_{1,-1}$ of the helicity density matrix of vector mesons produced in $e^+e^-$ annihilations; our predictions are in agreement with OPAL data on $\phi$, $D^*$ and $K^*$'s. New predictions are given for $\rho_{1,-1}$ of several mesons produced at large $x_E$ and small $p_T$ - i.e. collinear with the parent jet - in the annihilation of polarized $e^+$ and $e^-$; the results depend strongly on the elementary dynamics and allow further non trivial tests of the Standard Model.

Off-diagonal helicity density matrix elements for vector mesons produced in unpolarized and polarized $e^+e^-$ processes

M. Anselmino$^1$, M. Bertini$^2$, F. Caruso$^{3,4}$, F. Murgia$^5$ and P. Quintairos$^6$

$^1$ Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino, Via P. Giuria 1, 10125 Torino, Italy
Final state interactions and a coherent $q\bar{q}$ fragmentation process can give origin to off-diagonal matrix elements, $\rho_{1,-1}$, sizeably different from zero for vector mesons inclusively produced in $e^-e^+$ annihilations, \cite{1,2,3}. The usually adopted incoherent fragmentation scheme of a single independent quark leads to zero values for $\rho_{1,-1}$.

Predictions were given \cite{3} for vector mesons produced in unpolarized $e^-e^+$ annihilation processes, provided one selects mesons produced in two-jet events, with a large $x_E = E_\gamma / E_{beam}$ and a small transverse momentum, $p_T \simeq 0$. Such predictions have been confirmed by some experimental results from LEP \cite{4}. Subsequently \cite{5}, numerical estimates of $\rho_{1,-1}(V)$ in the coherent fragmentation of $q\bar{q}$ pairs in polarized $e^-e^+ \rightarrow q\bar{q} \rightarrow V X$ events were given.

The general expression of the helicity density matrix $\rho(V)$, which describes the polarization state of $V$, is \cite{1,2,3}:

$$
\rho_{\lambda_V,\lambda_V}(V) = \frac{1}{N_V} \sum_{q, x, \lambda'_q} D_{\lambda_V,\lambda_X;\lambda_q,\lambda'_{\bar{q}}} \times \rho_{\lambda_q,\lambda_{\bar{q}};x_{\lambda'_q},x'_{\lambda'_{\bar{q}}}}(q\bar{q}) D_{\lambda_V,\lambda_X;\lambda_q,\lambda'_{\bar{q}}}' ;
$$
is such that $\text{Tr} (\rho) = 1$ and the $D's$ are unknown non-perturbative helicity amplitudes for the fragmentation process. The matrix elements of $\rho \ (q\bar{q})$ depend on the elementary dynamics; for unpolarized initial beams we have [3]

$$\rho_{\lambda_q,\lambda_{\bar{q}};\lambda_q',\lambda_{\bar{q}}'}^{\text{unpol}} (q\bar{q}) = \frac{1}{4N_{q\bar{q}}^{\text{unpol}}} \sum_{\lambda_e=\ldots,\lambda_{e'+} \ldots} M_{\lambda_q,\lambda_{\bar{q}};\lambda_{e'},\lambda_{e'+}} \cdot (q\bar{q})^*,$$

where $N_{q\bar{q}}^{\text{unpol}}$ is an arbitrary factor.

In the case of polarized initial leptons Eq. (1) can be generalized into [5]:

$$\rho_{\lambda_q,\lambda_{\bar{q}};\lambda_q',\lambda_{\bar{q}}'}^{\text{pol}} (q\bar{q}) = \frac{1}{4N_{q\bar{q}}^{\text{pol}}} [(1 + \cos \alpha_-)(1 + \cos \alpha_+)]$$

$$\times M_{\lambda_q,\lambda_{\bar{q}};\lambda_{q'};\lambda_{\bar{q}}'} \times M_{\lambda_{q'};\lambda_{\bar{q}}'} \times M_{\lambda_{q'};\lambda_{\bar{q}}'} \times e^{i(\beta_- + \beta_+)} (\sin \alpha_- \sin \alpha_+)] ;$$

the $M'$s are the helicity amplitudes for the annihilation process. $\alpha_-$ and $\beta_-$ ($\alpha_+$ and $\beta_+$) are respectively the polar and azimuthal angle...
of the $e^- (e^+)$ spin vectors; we have chosen $xz$ as the scattering plane with $e^- (e^+)$ moving along the positive (negative) direction of $z$-axis. Let us remark that in the incoherent fragmentation scheme, all the off-diagonal elements of $\rho (V)$ vanish [2].

The helicity amplitudes for the fragmentation process, $D$, are unknown but, at least in some selected kinematical regions, it is possible to exploit angular momentum conservation to obtain further information [3]. The non vanishing off-diagonal elements of $\rho (V)$ are related to the values of the off-diagonal elements of $\rho (q\bar{q})$, in $e^- e^+$ annihilation processes, by

$$\rho_{1,-1}(V) \approx [1 - \rho_{0,0}(V)] \rho_{+-;--}(q\bar{q}) ;$$

both $\rho_{1,-1}(V)$ and $\rho_{0,0}(V)$ can be measured.

Computing $\rho_{+-;--}(q\bar{q})$ at LEP energies ($\sqrt{s} = M_Z$) where the weak interaction dominates, one gets, for unpolarized initial beams ($e^- e^+$) [3]:

$$\rho_{+-;--}(q\bar{q}) \approx \frac{1}{2} \frac{c_V^2 - c_A^2}{c_V^2 + c_A^2} \frac{\sin^2 \theta}{1 + \cos^2 \theta} ;$$

$\theta$ is the angle between the incoming negative lepton and the outgoing quark in the $e^- e^+$ c.m. frame and the $c's$ are the usual Standard Model coupling constants [6]. Our theoretical predictions [3] and the experimental data from Ref. [4] are given below. Some data from DELPHI [7] do not confirm the agreement.

<table>
<thead>
<tr>
<th>OPAL Collab.</th>
<th>Theory</th>
</tr>
</thead>
</table>

...sbfisica.org.br/./res60/ 4/10
In the case of polarized initial lepton beams one obtains [5]:

\[
\rho_{++,-;+-}^{pol}(q\bar{q}) = \frac{1}{4N_{q\bar{q}}^{pol}} \left[ (1 + \cos^2 \theta)F_{1,q}^{pol} + \cos \theta F_{2,q}^{pol} + \sin^2 \theta F_{3,q}^{pol} \right] 
\]

(5)

\[
\rho_{+-,-;--}^{pol}(q\bar{q}) = \frac{1}{4N_{q\bar{q}}^{pol}} \left[ (1 + \cos^2 \theta)(F_{4,q}^{pol} + iF_{5,q}^{pol}) + \cos \theta (F_{6,q}^{pol} + iF_{7,q}^{pol}) + \sin^2 \theta (F_{8,q}^{pol} + iF_{9,q}^{pol}) \right] 
\]

(6)

with

\[
N_{q\bar{q}}^{pol} = (1 + \cos^2 \theta)F_{10,q}^{pol} + \cos \theta F_{11,q}^{pol} + \sin^2 \theta F_{12,q}^{pol} . 
\]

(7)

The complete list of all the \( F_{i,q}^{pol} \) is given in Ref. [5]; these functions depend on the Standard Model coupling constants and on the
polarization angles $\alpha_\pm$ and $\beta_\pm$. Considering all independent polarizations directions we get 36 possible initial spin states; many of them will lead to the same value of $\rho_{++--}^{pol}(q\bar{q})$ and it is convenient to group them into 9 cases. The complete description of these groups is again given in Ref. [5].

We show our numerical results for $\rho_{++--}^{pol}(q\bar{q})$ in a set of figures. We give results only for those cases which strongly differ from the unpolarized case and have such peculiar features which would make a measurement of $\rho_{1,-1}(V)$ in agreement with them an unquestionable test of our approach. A complete discussion is presented in Ref. [5].

The set of papers [1],[2],[3] and [5] gives a complete study of the off-diagonal helicity density matrix element $\rho_{1,-1}(V)$ of vector mesons produced in $e^+e^-$ annihilations into two jets, selecting vector mesons with a large energy fraction (say $x_E \gtrsim 0.5$) and small transverse momentum ($p_T/(x_E \sqrt{s}) \ll 1$) inside one of the jets. Initially, the idea was given in Refs. [1] and [2], and the first numerical predictions, given in Ref. [3], have been confirmed by some experimental data [4]. In [5] it was considered the most general case of polarized $e^+$ and $e^-$. A discussion of $\rho_{1,-1}(V)$ for vector mesons produced in different processes can be found in Ref. [8].

Nowadays, there is no operating $e^+e^-$ collider with polarized beams but the future generations of linear colliders are being planned and our results might indicate good reasons to consider polarization options.

Acknowledgements P. Q. is grateful to FAPESP of Brazil for financial support. M. B. and P. Q. would like to thank the Department of Theoretical Physics of Università di Torino where this work was initiated. The work of M. B. is supported in part by the EU Fourth
Framework Programme 'Training and Mobility of Researchers',
Network 'Quantum Chromodynamics and the Deep Structure of
Elementary Particles', contract FMRX-CT98-0194 (DG 12 - MIHT).

\[ \text{Re} \beta_{4,-4}^\text{pe}(u) \]

\[ \theta \text{ [degrees]} \]

\[ \text{Re} \beta_{4,-4}^\text{pe}(u) \]

\[ \text{Re} \beta_{4,-4}^\text{pe}(u) \]

\[ \text{Re} \beta_{4,-4}^\text{pe}(u) \]

\[ \text{Re} \beta_{4,-4}^\text{pe}(u) \]

\[ \text{Re} \beta_{4,-4}^\text{pe}(u) \]

\[ \text{Re} \beta_{4,-4}^\text{pe}(u) \]

\[ \text{Re} \beta_{4,-4}^\text{pe}(u) \]

\[ \text{Re} \beta_{4,-4}^\text{pe}(u) \]
Off-diagonal helicity density

\[ \text{Im} \rho_{+-,-+}^{u}(\theta) \]

\[ \text{Im} \rho_{+-,-+}^{d}(\theta) \]

\[ \theta \text{ [degrees]} \]