Fundamentals of Magnetism

Part II

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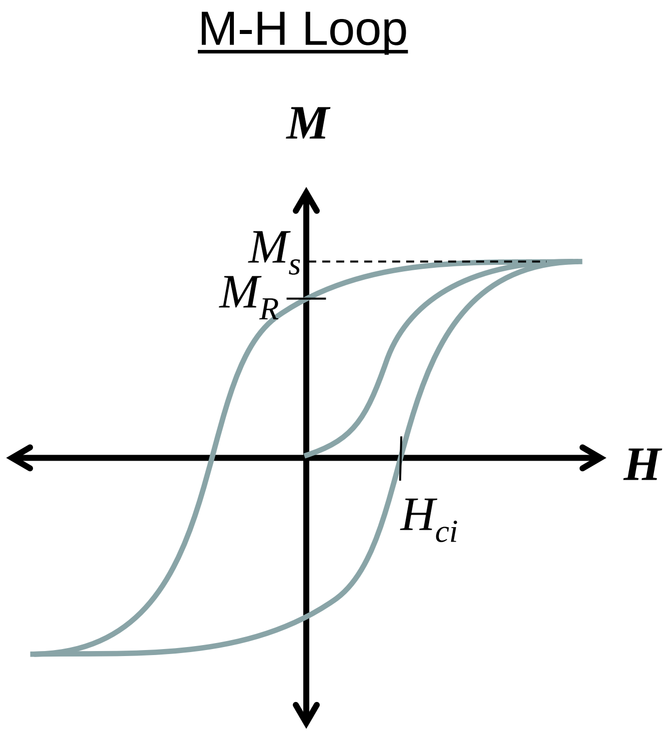
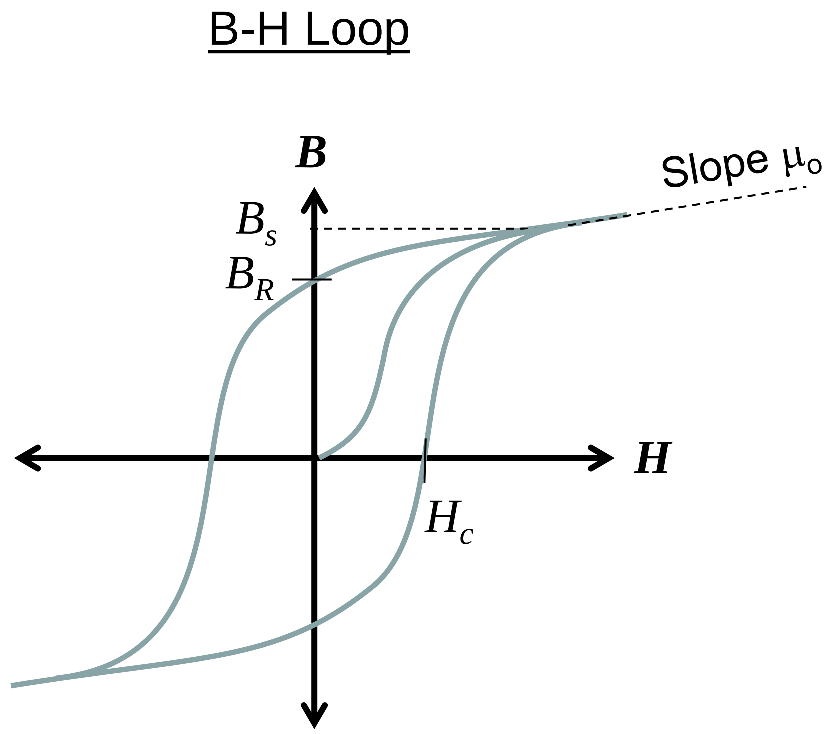
Oregon State University

Real Magnetic Materials, Bulk Properties



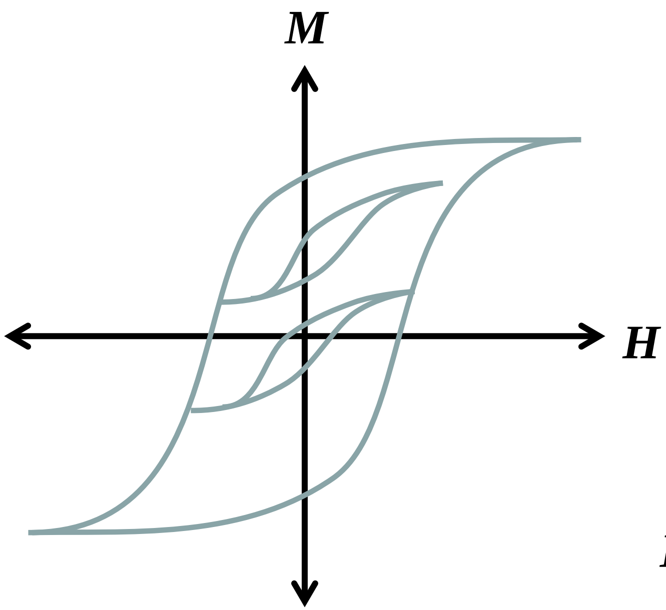


M_S - Saturation magnetization
H_{ci} - Intrinsic coercivity
M_R - Remanent magetization

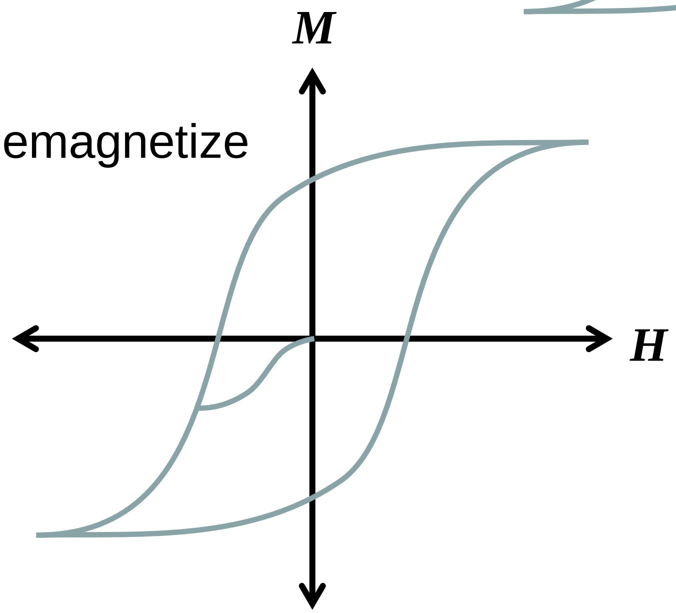
B_s - Saturation flux density
B_R - Remanent flux density
H_c - Coercive field, coercivity

Note: these two figures each present the same information because $B=\mu_o(H+M)$

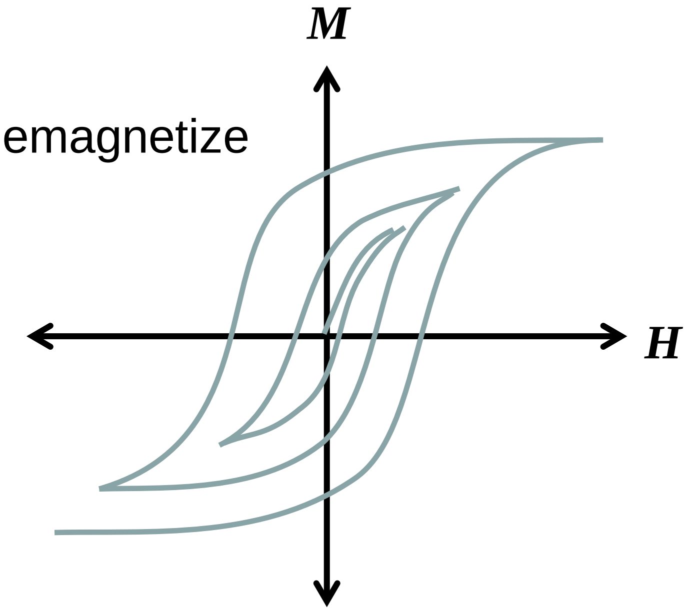
Minor Loops



DC demagnetize

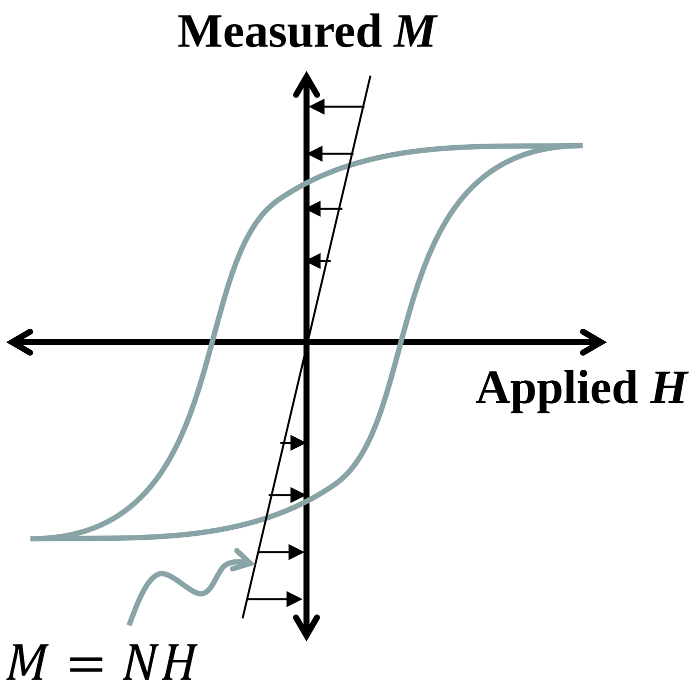


AC demagnetize

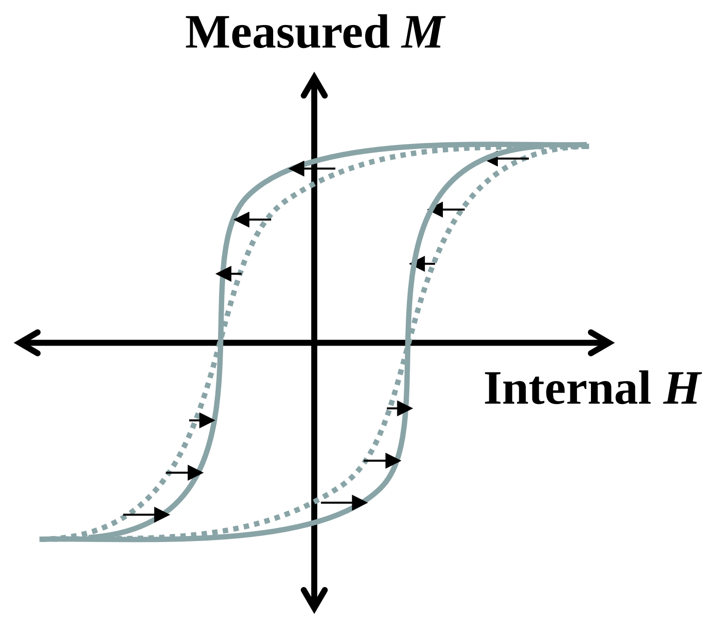


Effect of Demag. Fields on Hysteresis Loops

Measured loop



Actual material loop



Types of Magnetism

Diamagnetism – no atomic magnetic moments

Paramagnetism – non-interacting atomic moments

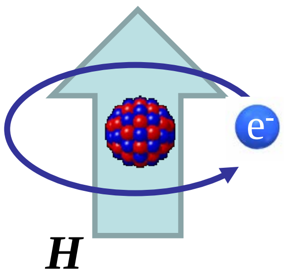
Ferromagnetism – coupled moments

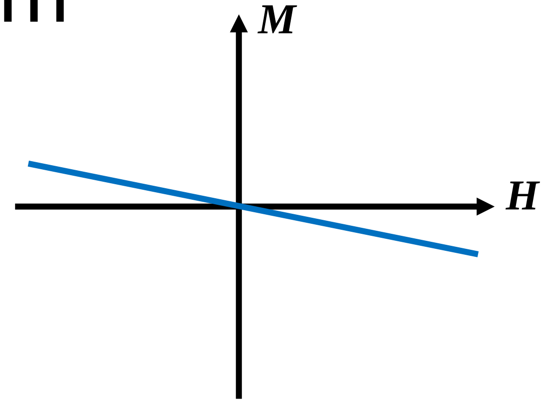
Antiferromagnetism – oppositely coupled moments

Ferrimagnetism – oppositely coupled moments of different magnitude

Diamagnetism

- Atoms without net magnetic moment





χ is small and negative

Diamagnetic substances:

Ag		-1.0×10^{-6}
Be		-1.8×10^{-6}
Au		-2.7×10^{-6}
H_2O*		-8.8×10^{-6}
NaCl		-14×10^{-6}
Bi		-170×10^{-6}
Graphite		-160×10^{-6}
Pyrolitic	(⊥)	-450×10^{-6}
graphite	(‖)	-85×10^{-6}

Atomic number

Orbital radius

Atomic density

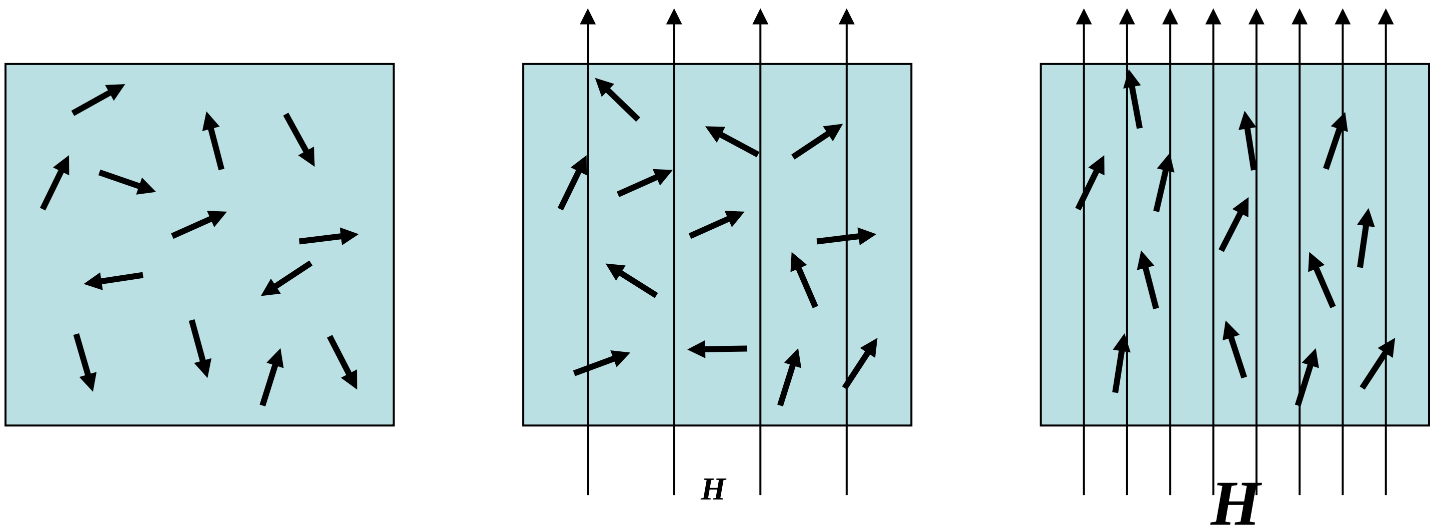
$$\chi = -\frac{N_A \rho}{W_A}\frac{\mu_0 Z e^2 r^2}{6 m_e}$$

* E.g. humans, frogs, strawberries, etc.
See: http://www.hfml.ru.nl/froglev.html

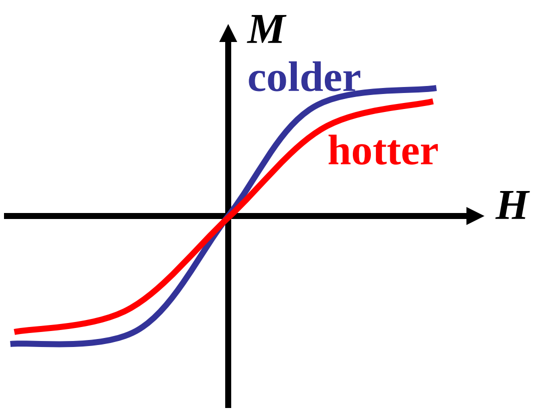
Paramagnetism

• Consider a collection of atoms with independent magnetic moments.

• Two competing forces: spins try to align to magnetic field but are randomized by thermal motion.



$\mu_0 mH << k_B T$ $\mu_0 mH > k_B T$

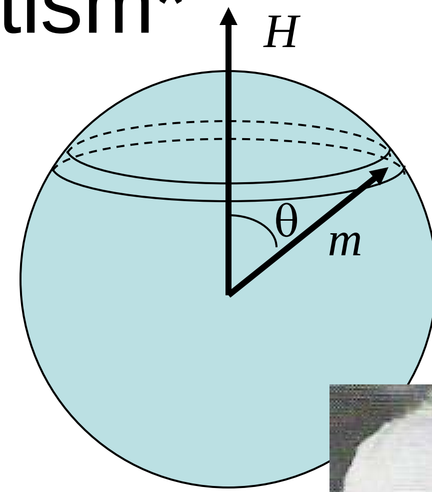


• Susceptibility is small and positive

• Susceptibility decreases with temperature.

• Magnetization saturates (at low T or very large H)

Langevin Theory of Paramagnetism*



- Energy of a magnetic moment, m, in a field, H

$$E = -\mu_0 \vec{m} \cdot \vec{H} = -\mu_0 mH \cos(\theta)$$

- Independent moments follow Boltzmann statistics

$$P(E) = e^{-E/k_B T} = e^{\mu_0 mH \cos(\theta)/k_B T}$$

- The "density of states" on the sphere is

$$dn = 2\pi r^2 \sin(\theta) d\theta$$

- So the probability of a moment pointing in direction θ to θ+dθ is

$$p(\theta) = \frac{e^{\mu_0 mH \cos(\theta)/k_B T} \sin(\theta)}{\int_0^\pi e^{\mu_0 mH \cos(\theta)/k_B T} \sin(\theta) d\theta}$$

Paul Langevin
(1872-1946)

- The magnetization of a sample with N moments/volume will be

$$M = Nm\langle \cos(\theta) \rangle = Nm\int_0^\pi \cos(\theta)\, p(\theta) d\theta = Nm \frac{\int_0^\pi e^{\mu_0 mH \cos(\theta)/k_B T} \cos(\theta) \sin(\theta) d\theta}{\int_0^\pi e^{\mu_0 mH \cos(\theta)/k_B T} \sin(\theta) d\theta}$$

- Which any fool can see is just:

$$M = Nm\left[\coth\left(\frac{\mu_0 mH}{k_B T}\right) - \frac{k_B T}{\mu_0 mH}\right]$$

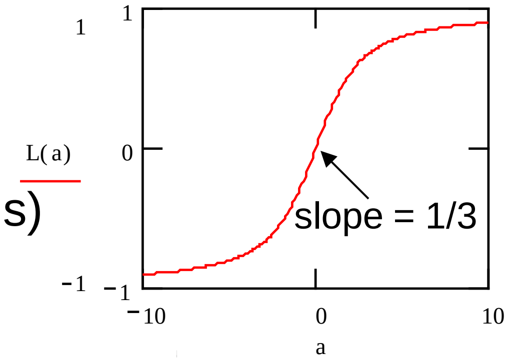
*Langevin, P., Annales de Chem. et Phys., 5, 70, (1905).

Curie's Law*

- Langevin paramagnetism:

$$M = Nm\left[\coth\left(\frac{\mu_0 mH}{k_B T}\right) - \frac{k_B T}{\mu_0 mH}\right]$$

$$L(a) = \coth(a) - \frac{1}{a}$$



Pierre Curie
(1859-1906)

- For $\mu_o mH << k_B T$ (most practical situations)

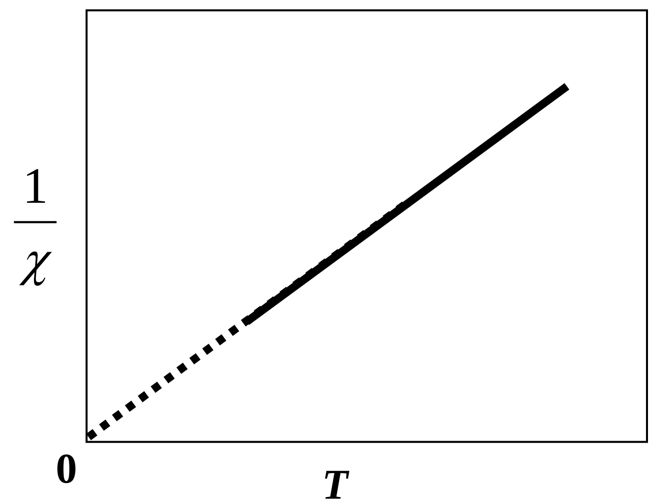
$$M = \mu_0 \frac{Nm^2}{3k_B T} H$$

- Which gives Curie's Law:

$$\chi = \frac{M}{H} = \mu_0 \frac{Nm^2}{3k_B T} = \frac{C}{T}$$

Curie constant

Curie's Law



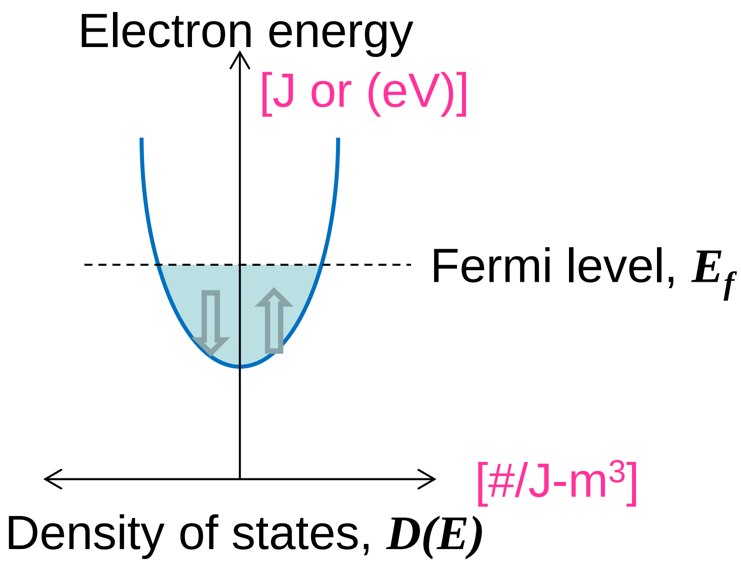
Note: quantization of the magnetic moment requires a correction to the above classical result which assumes all directions are allowed:

$$\chi = \frac{M}{H} = \mu_0 \frac{Ng^2 \mu_B^2 J(J+1)}{3J^2 k_B T} = \frac{C}{T}$$

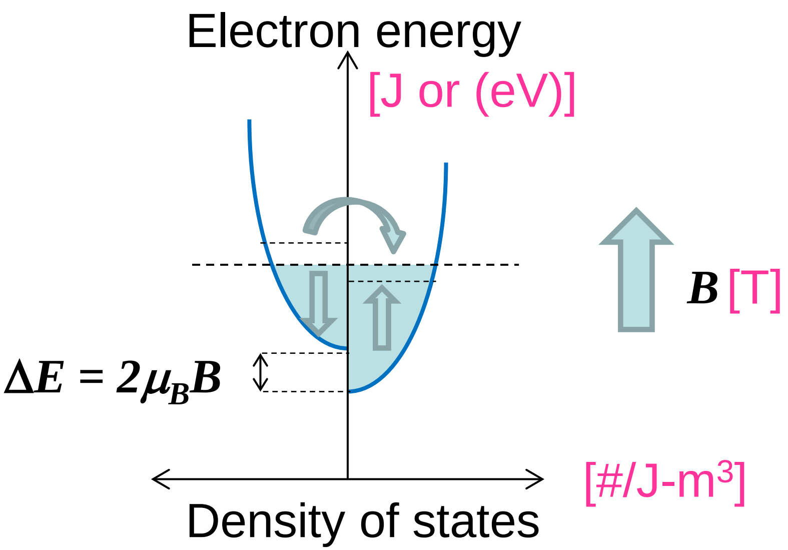
*Curie, P., Ann. Chem. Phys., **5,** 289 (1895)

Pauli Paramagnetism

<u>No magnetic field</u>



<u>In magnetic field, B</u>



$$M = \mu_B^2 B D(E_f)$$

Paramagnetic Substances

Paramagnetic susceptibility:

Sn	0.19×10^{-6}
Al	1.65×10^{-6}
O_2 (gas)	1.9×10^{-6}
W	6.18×10^{-6}
Pt	21.0×10^{-6}
Mn	66.1×10^{-6}

Liquid oxygen

Curie-Weiss* Law

- Consider <u>interactions</u> among magnetic moments:

$$E = -\mu_0 \vec{m} \cdot (\vec{H} + \alpha \vec{M})$$

where M represents the average orientation of the surrounding magnetic moments ("Mean field theory") and α is the strength of the interactions

Pierre Weiss (1865–1940)

- The Langevin equation then becomes:

$$M = Nm\left[\coth\left(\frac{\mu_0 m(H + \alpha M)}{k_B T}\right) - \frac{k_B T}{\mu_0 m(H + \alpha M)}\right]$$

- Again for $\mu_o mH << k_B T$

$$M = \mu_0 \frac{Nm^2}{3k_B T}(H + \alpha M)$$

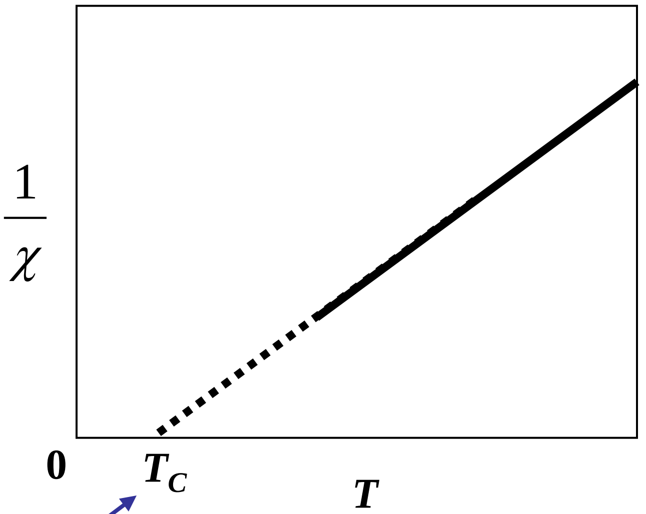
- Which gives the Curie-Weiss Law:

$$\chi = \frac{M}{H} = \frac{C}{T - \alpha C} = \frac{C}{T - T_C}$$

$$T_C = \mu_0 \frac{\alpha N m^2}{3k_B}$$

Curie Temperature

Curie-Weiss Law

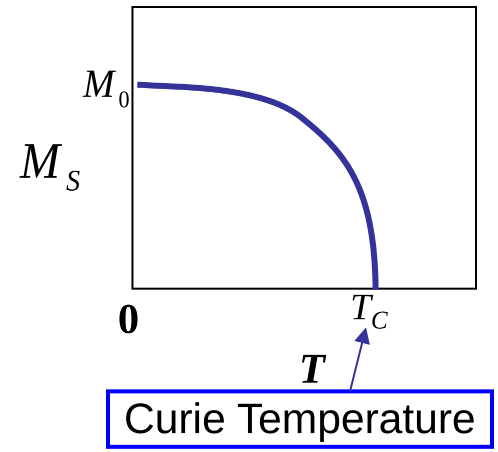


*Weiss, P., J. de Phys., **6,** 661 (1907)

Weiss Theory* of Ferromagnetism

- Starting with the Langevin equation with “mean field” interactions

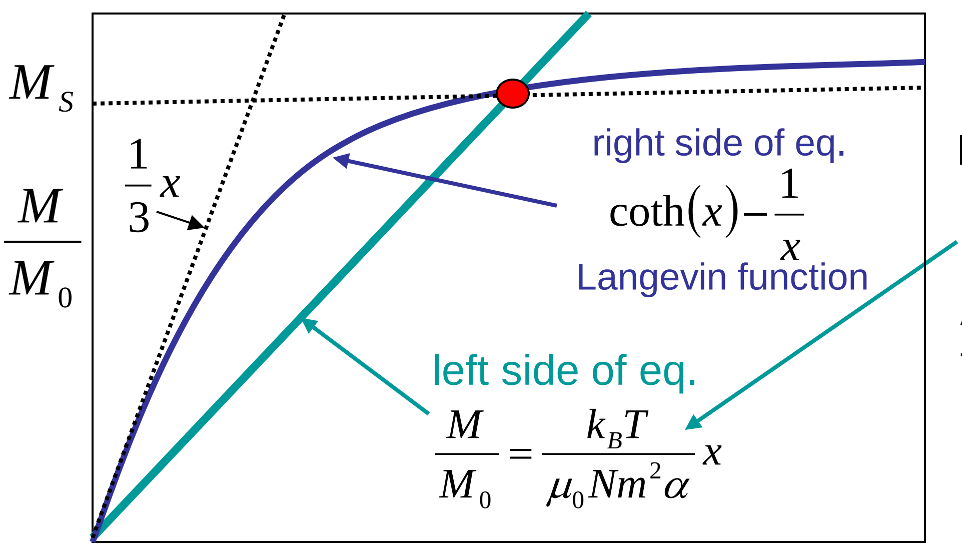
$$M = Nm\left[\coth\left(\frac{\mu_0 m(H+\alpha M)}{k_B T}\right) - \frac{k_B T}{\mu_0 m(H+\alpha M)}\right]$$



- Set the external field to zero, $H=0$

$$M = Nm\left[\coth\left(\frac{\mu_0 m(\alpha M)}{k_B T}\right) - \frac{k_B T}{\mu_0 m(\alpha M)}\right]$$

- This can be solved for M graphically:



$$x = \frac{\mu_0 m \alpha M}{k_B T}$$

Plot both sides of the equation as a function of

$$x = \frac{\mu_0 m \alpha M}{k_B T}$$

Intersection is <u>spontaneous magnetization</u>, M_s

Note: line with slope proportional to T

Asymptote of Langevin function is 1/3 so the only solution for M_S is zero when

$$\frac{k_B T}{\mu_0 N m^2 \alpha} > \frac{1}{3}$$

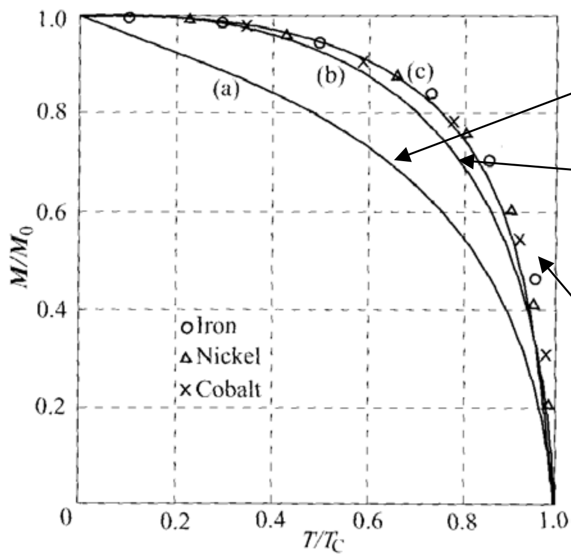
Spontaneous magnetization decreases with temperature and vanishes at:

$$T_C = \frac{\mu_0 N m^2 \alpha}{3 k_B}$$

*Weiss, P., J. de Phys., **6,** 661 (1907)

Weiss Theory of Ferromagnetism

- Fits data very well!
- Again, corrections need to be made for quantization of magnetic moments



(a) Classical result using Langevin function

(b) Quantum result using Brilloun function

$$M = Nm\tanh\left(\frac{\mu_0 m(H+\alpha M)}{k_B T}\right)$$

(c) Measurements

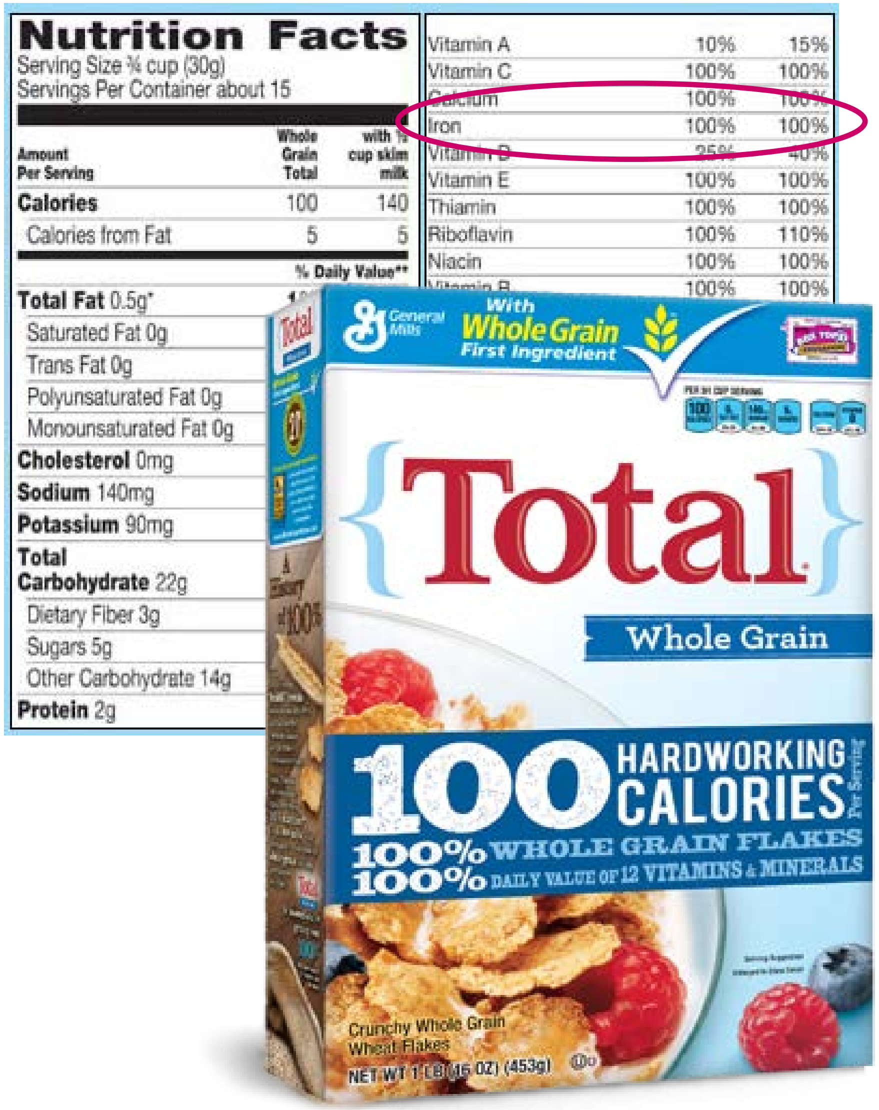
Points to remember:

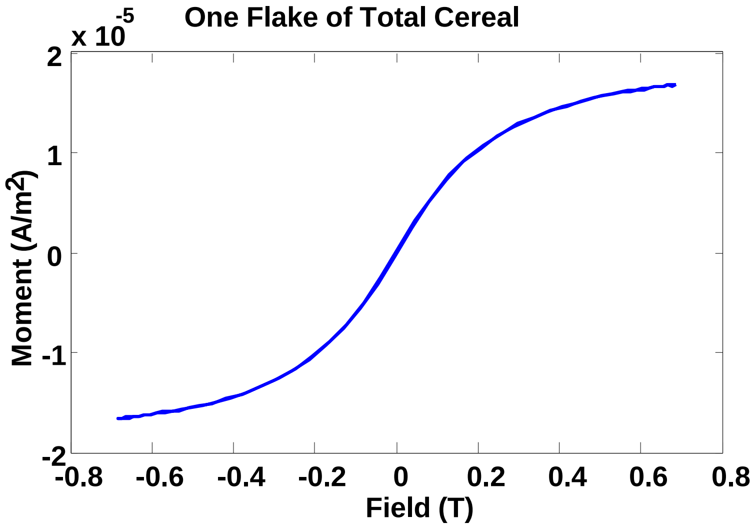
- Ferromagnets have spontaneous magnetization, M_S even in zero field
- M_S goes to zero at the Curie temperature, T_C
- Above T_C material is paramagnetic.

Ferromagnetic Materials

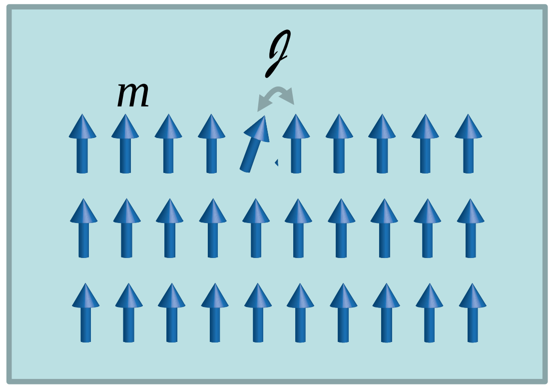
Material:	M_s [A/m]	T_C [°C]
Ni	0.49×10^6	354
Fe	1.7×10^6	770
Co	1.4×10^6	1115
Gd		19
Dy		-185
NdFeB	1.0×10^6	310
NiFe	$\sim 0.8 \times 10^6$	447
FeCoAlO	1.9×10^6	

Ferromagnetic Breakfast

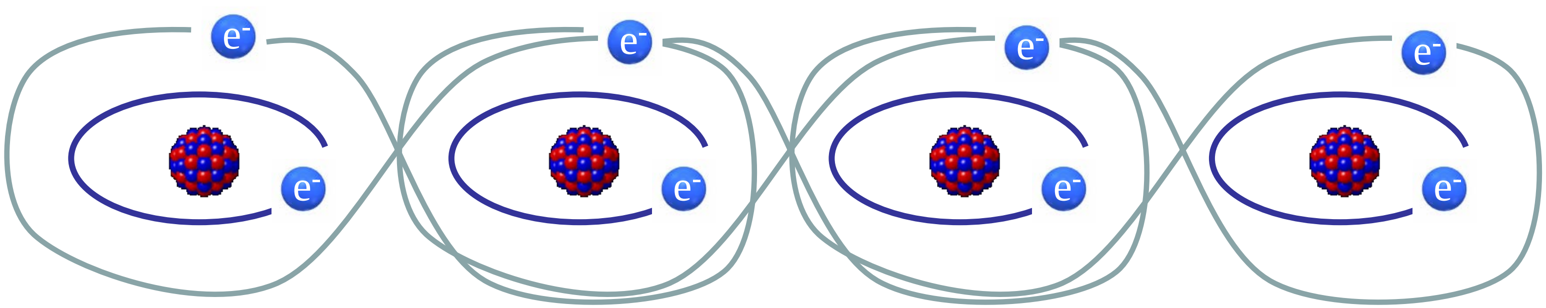




Ferromagnetic Exchange Interaction



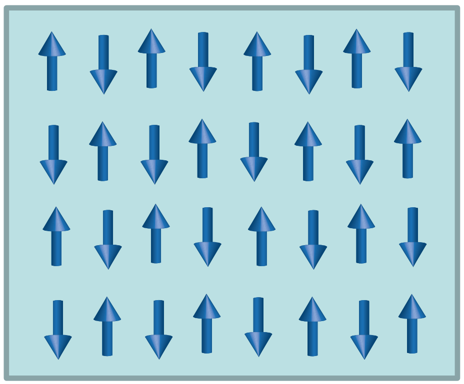
$$E_{m_i, m_{i+1}} = -\mu_0 \mathcal{J} m^2 \cos(\Delta\theta)$$



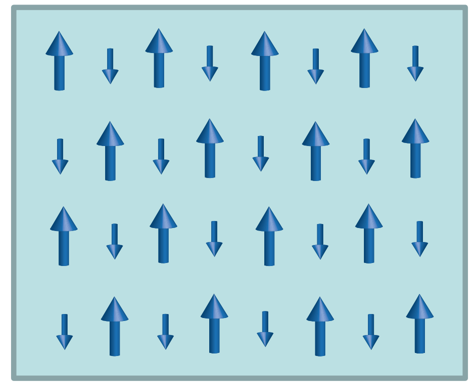
Antiferromagnetic and Ferrimagnetic Materials

Negative exchange coupling:

Antiferromagnet

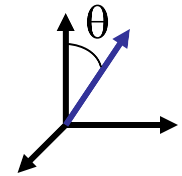


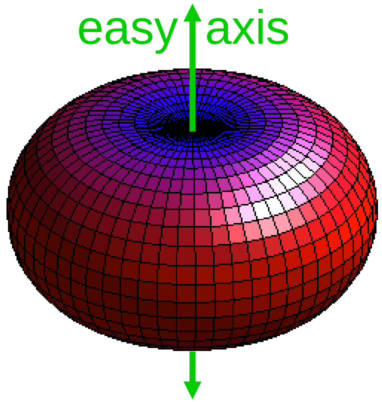
Ferrimagnet



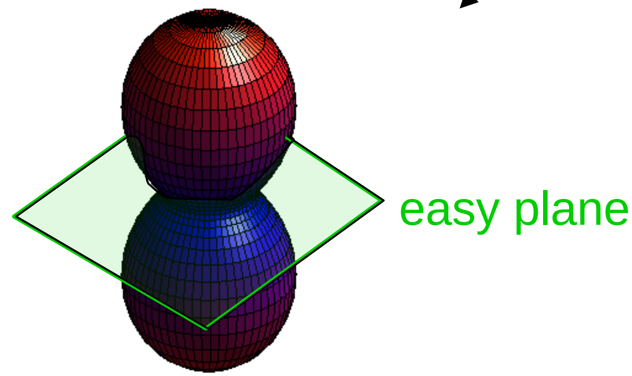
Crystalline Anisotropy

- **Uniaxial:** $E = K_u \sin^2\theta + K_2 \sin^4\theta + \ldots$



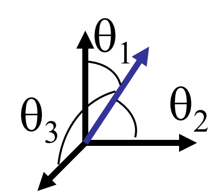


$K_u > 0$

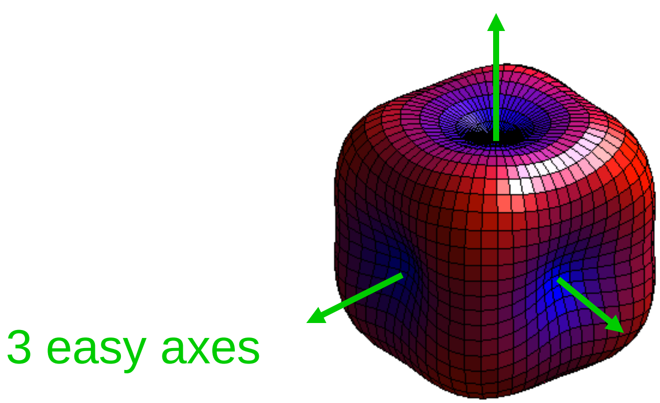


$K_u < 0$

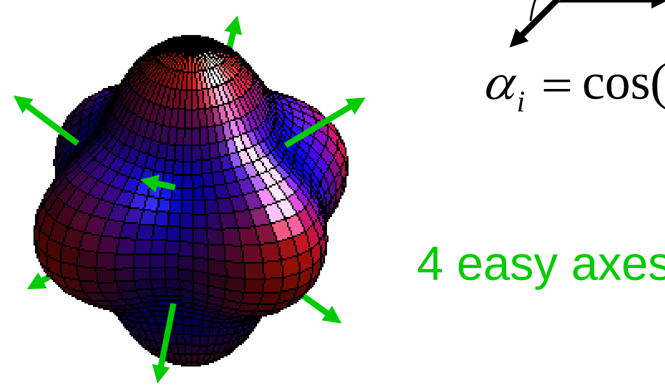
- **Cubic:** $E = K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_1^2\alpha_3^2) + K_2(\alpha_1^2\alpha_2^2\alpha_3^2) + \ldots$



$\alpha_i = \cos(\theta_i)$



$K_1 > 0$



$K_1 < 0$

Crystalline Anisotropies of Some Magnetic Materials

Material	Structure	K_1 [J/m^3] x10^5	K_2 [J/m^3] x10^5
Cobalt	hcp	4.1	1.5
Iron	bcc	0.48	-0.1
Nickel	fcc	-0.045	-0.023
$SmCo_5$		170	

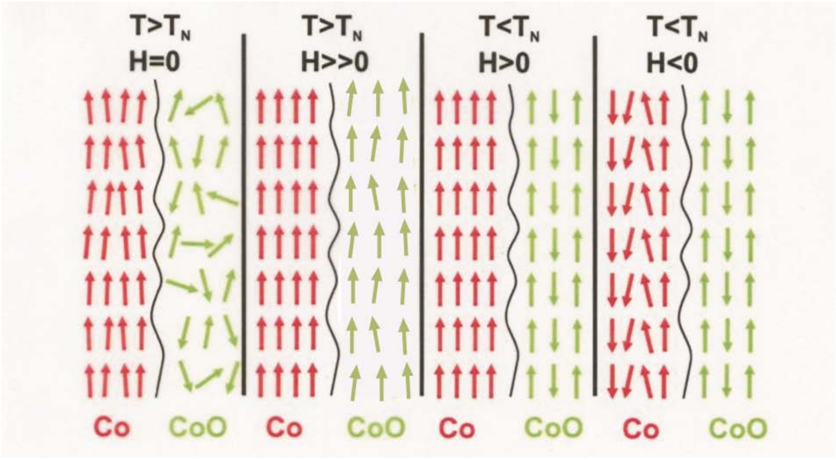
Other sources of anisotropy

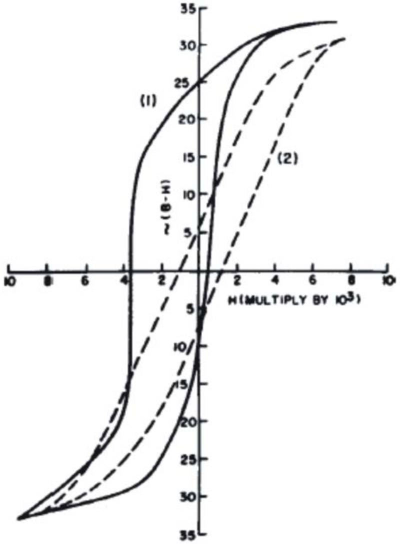
Induced

- Heat in a field, stress, plastic deformation (e.g., rolling), etc.

Exchange anisotropy

- coupling between FM and AFM materials





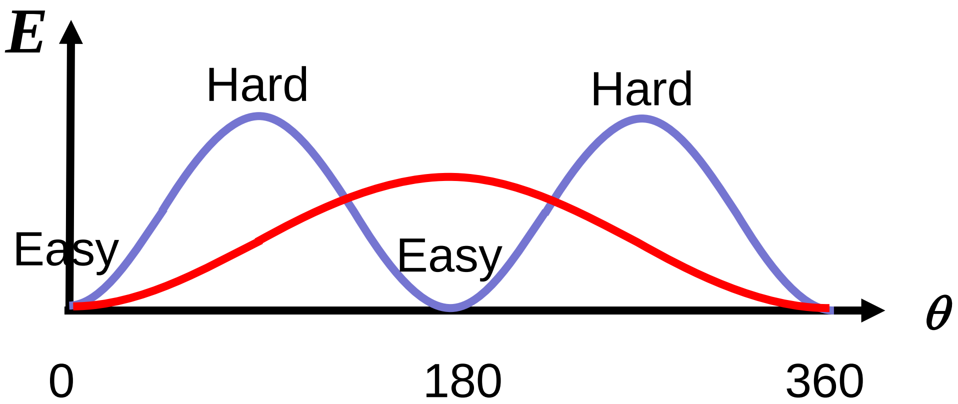
Stoner-Wohlfarth Theory*

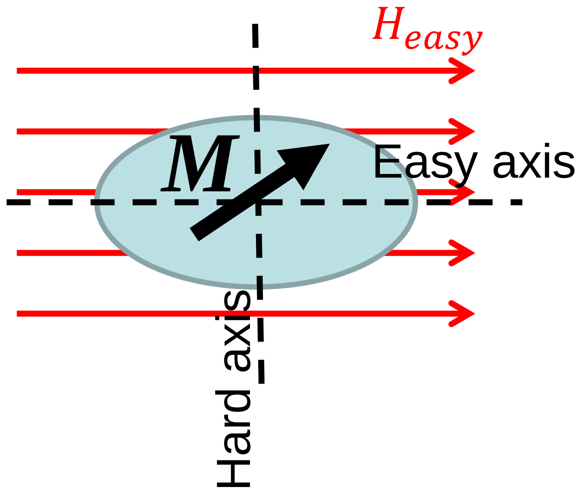
Anisotropy energy

$$E_{anis} = K_u V sin^2(\theta)$$

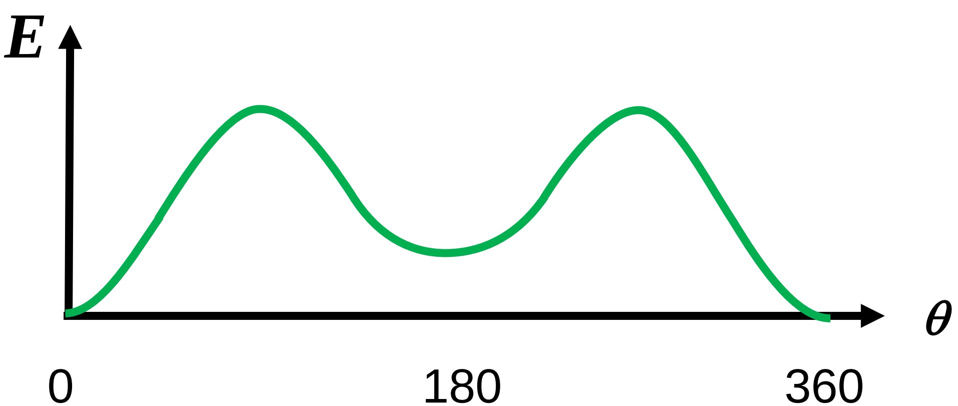
Magnetostatic energy

$$E_{ms} = H_{easy} M_s V cos\,(\theta)$$





Total energy (Magnetic field along easy axis)



* E. Stoner and P. Wohlfarth, "A Mechanism of Magnetic Hystereisis in Heterogeneous Alloys", IEEE Trans. Magn., **27**, 3475, 1991

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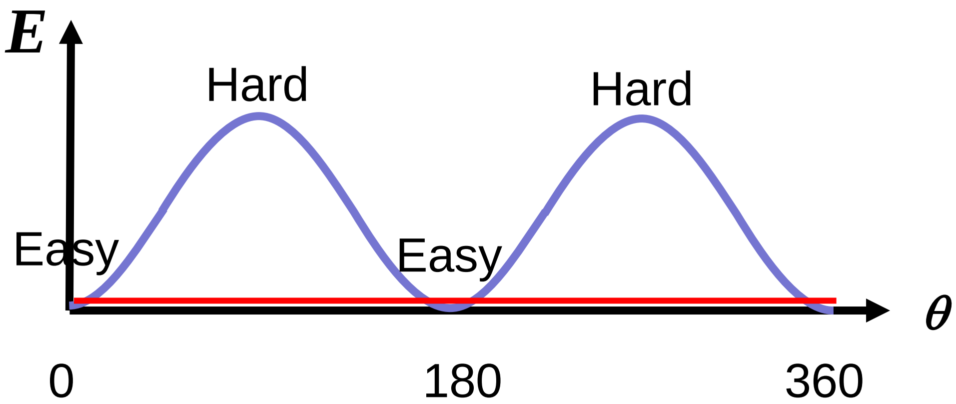
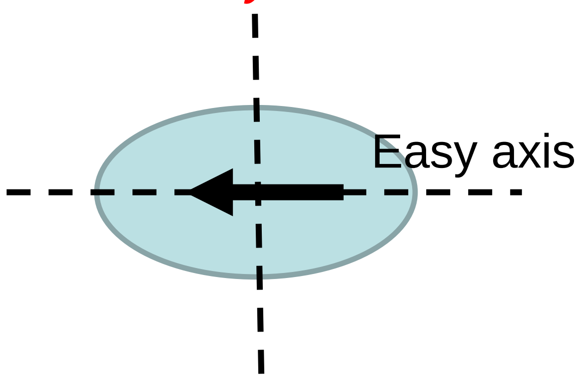
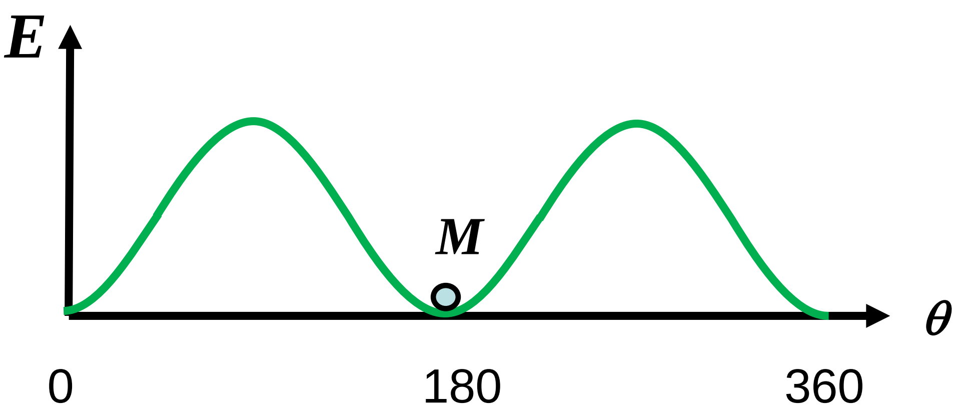
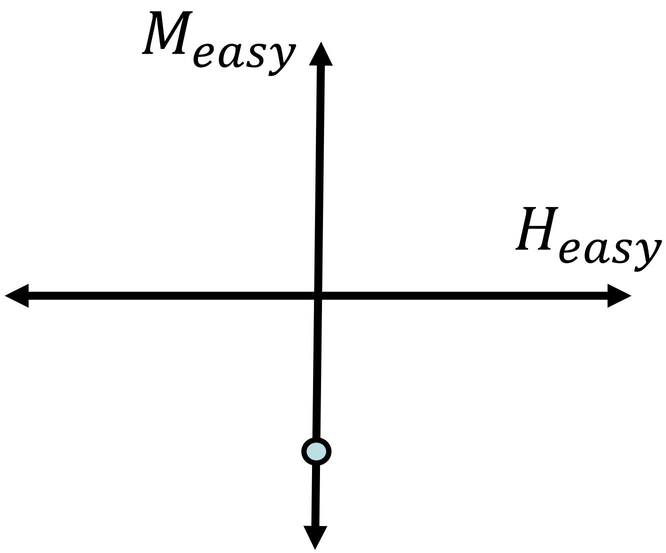


$H_{easy} = 0$



Total energy (Magnetic field along easy axis)





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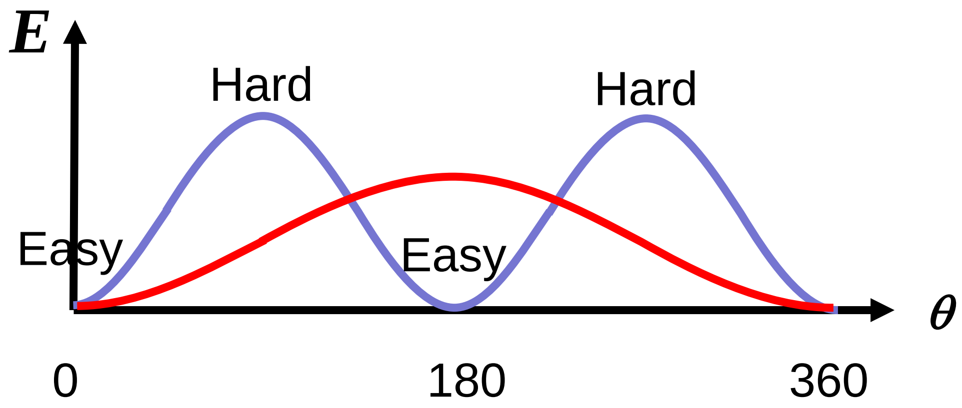
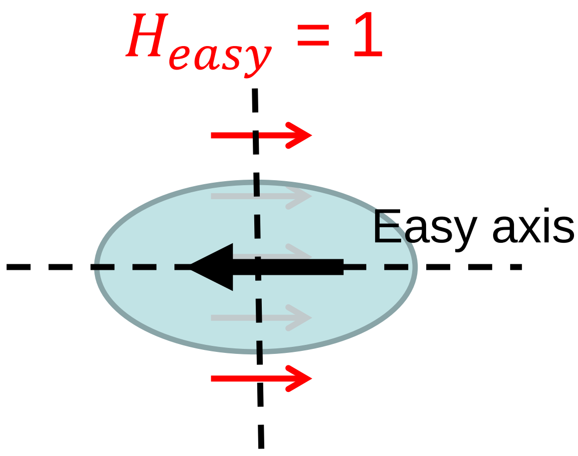
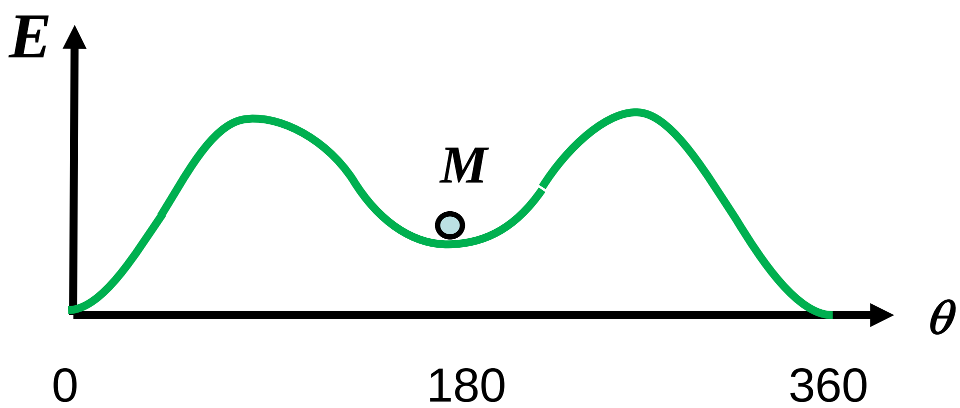
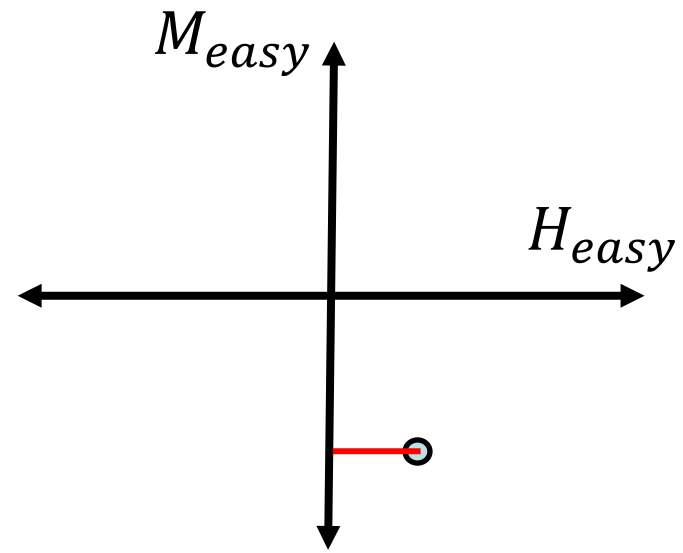




Total energy (Magnetic field along easy axis)





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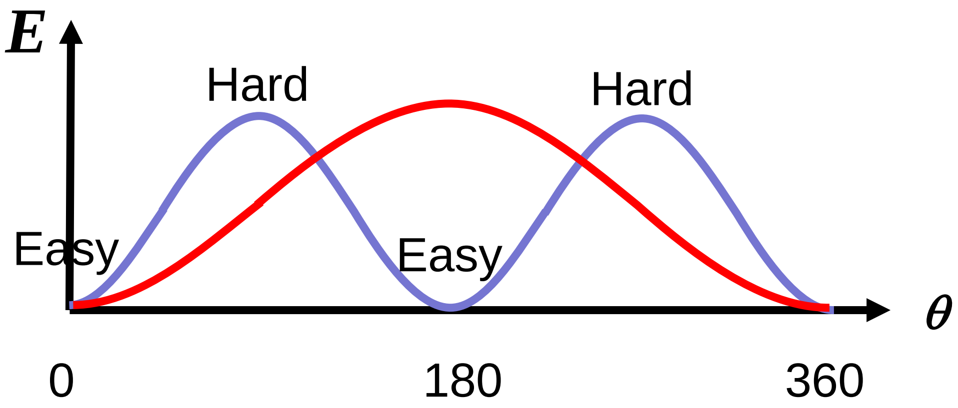
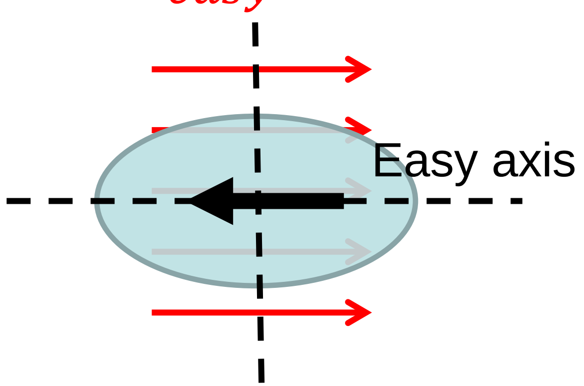
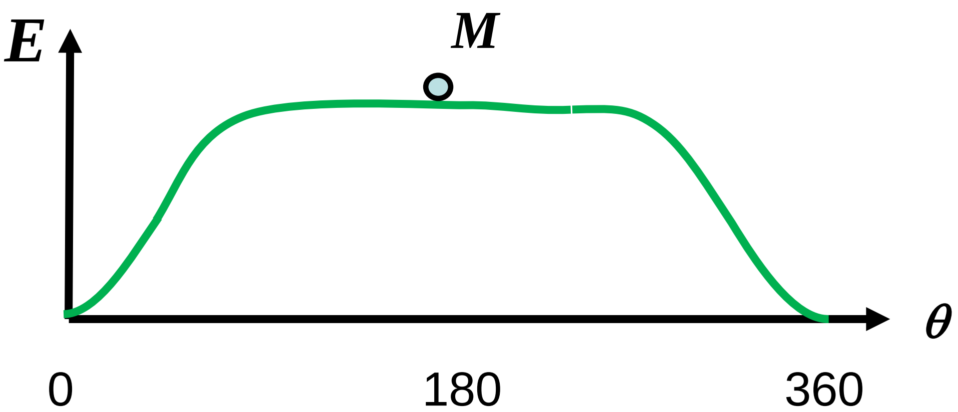
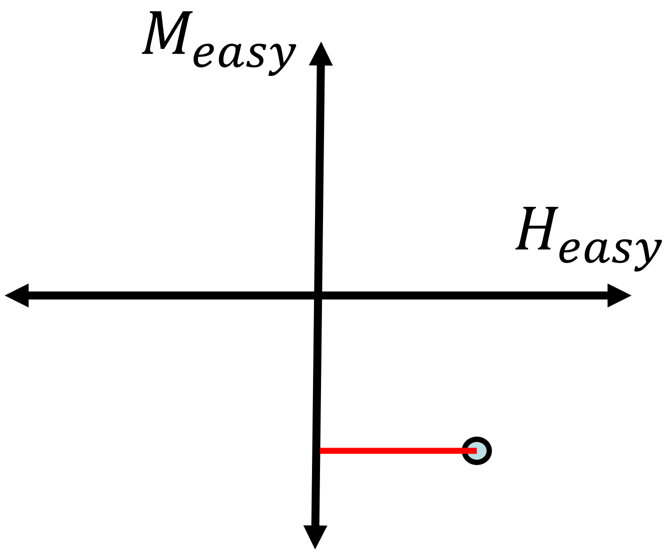




Total energy (Magnetic field along easy axis)





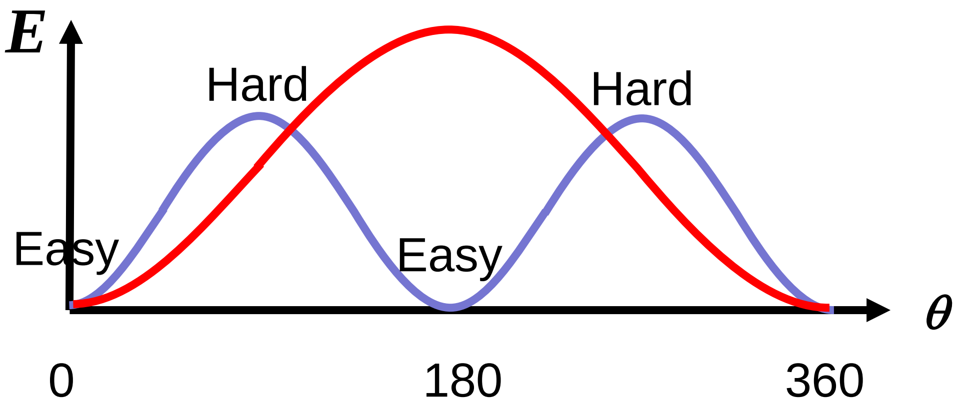
Stoner-Wohlfarth Theory*

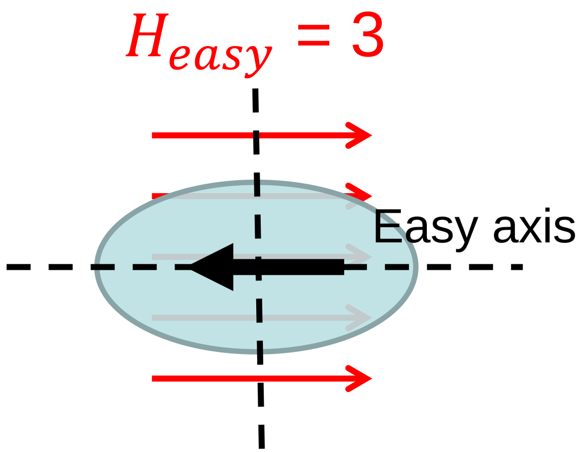
Anisotropy energy

$$E_{anis} = K_u V sin^2(\theta)$$

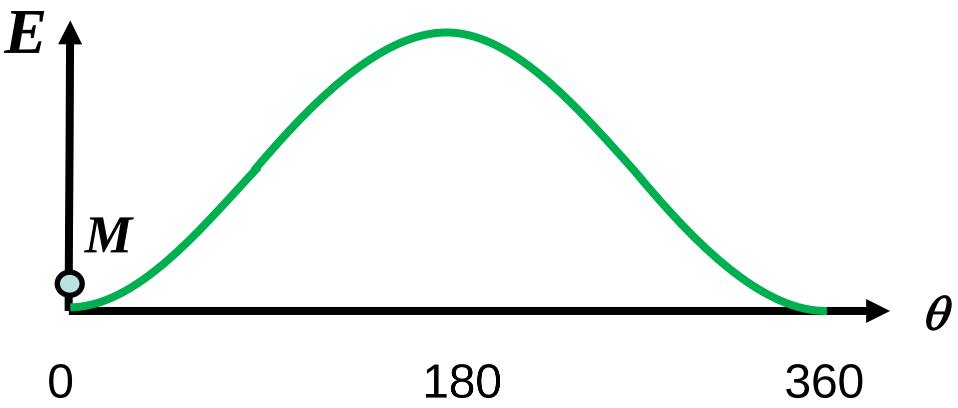
Magnetostatic energy

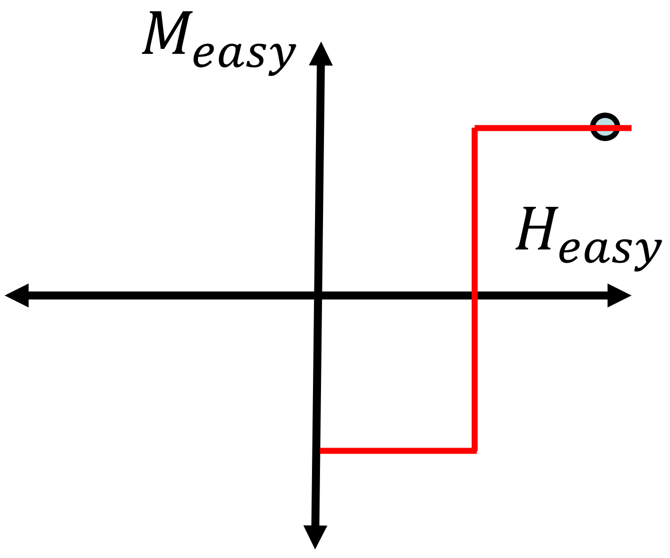
$$E_{ms} = H_{easy} M_s V cos\,(\theta)$$





Total energy (Magnetic field along easy axis)





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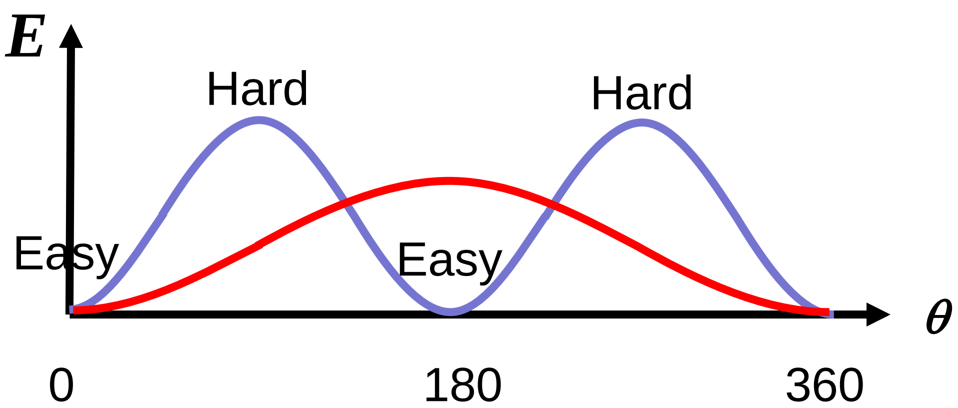
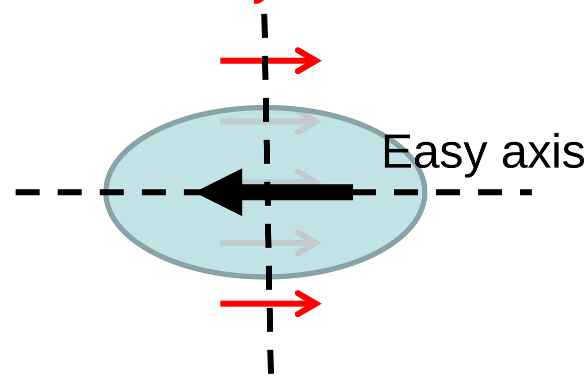
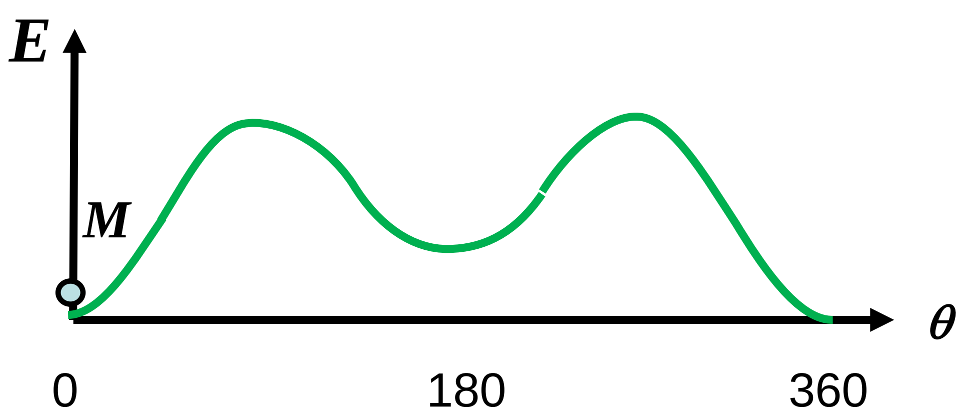
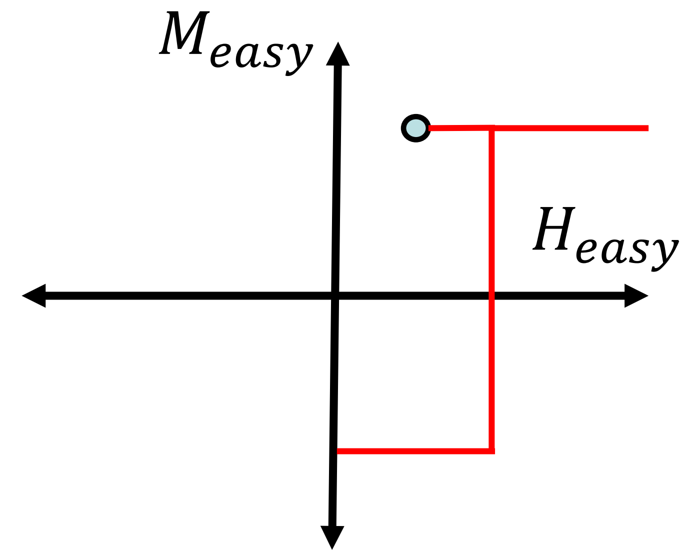


H_{easy} = 1



Total energy (Magnetic field along easy axis)





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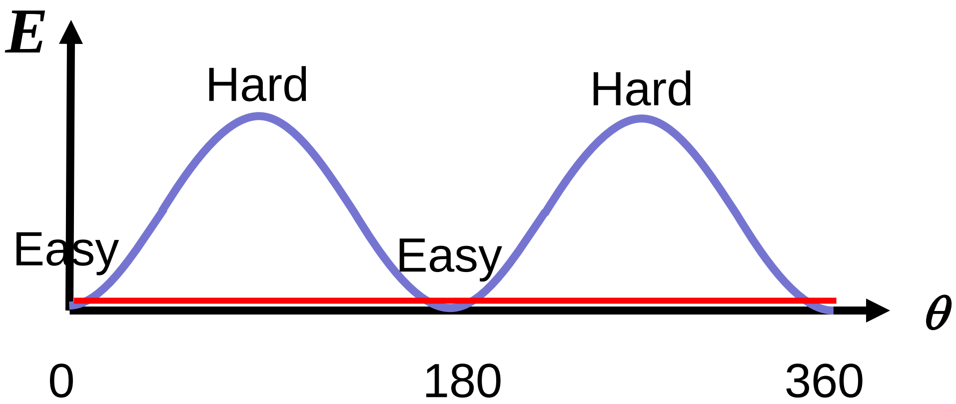
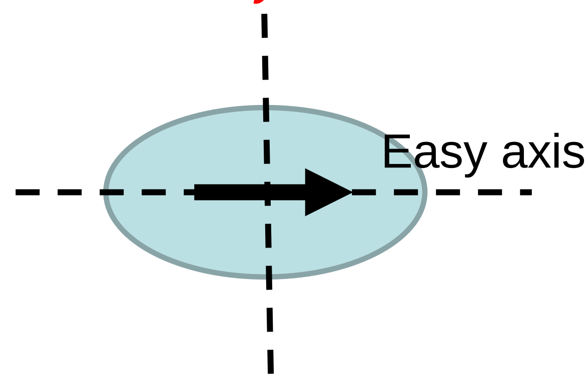
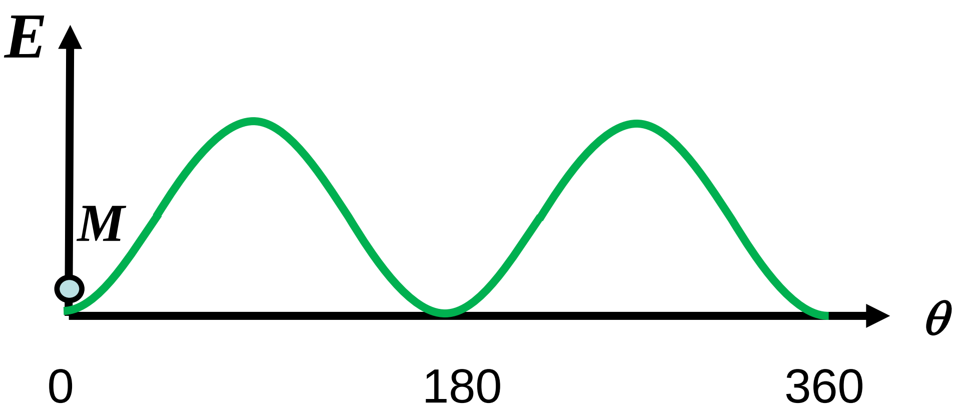
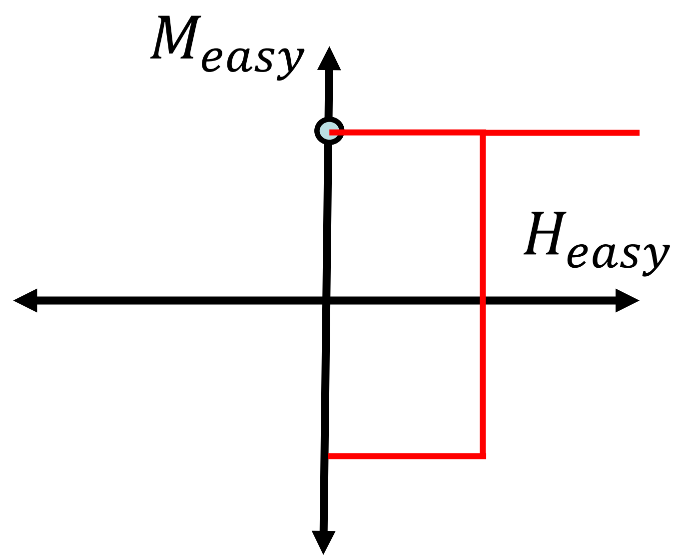


H_{easy} = 0



Total energy (Magnetic field along <u>easy axis</u>)





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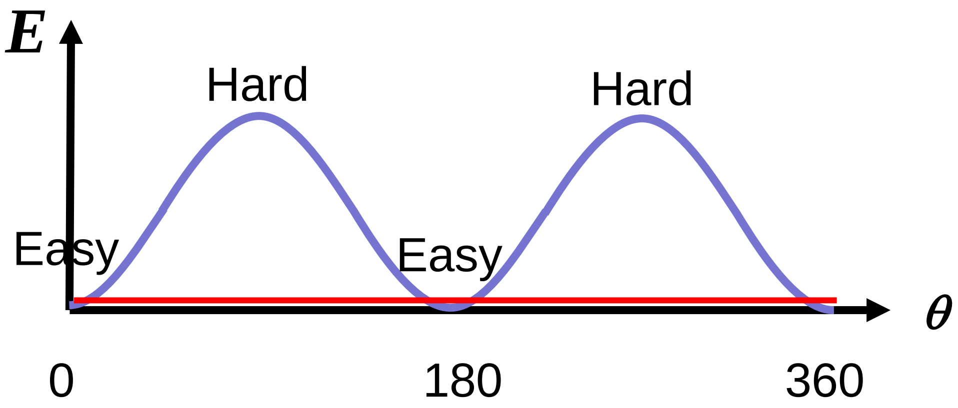
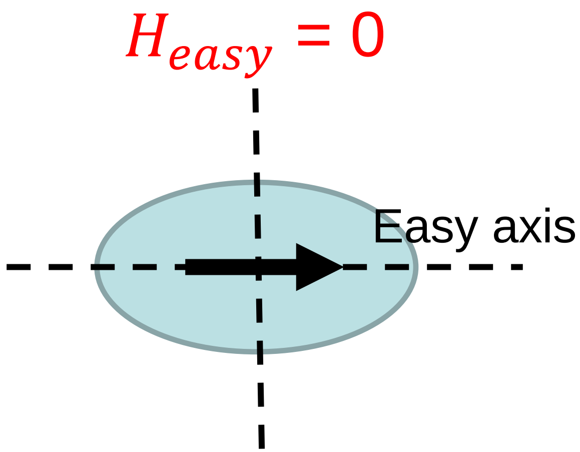
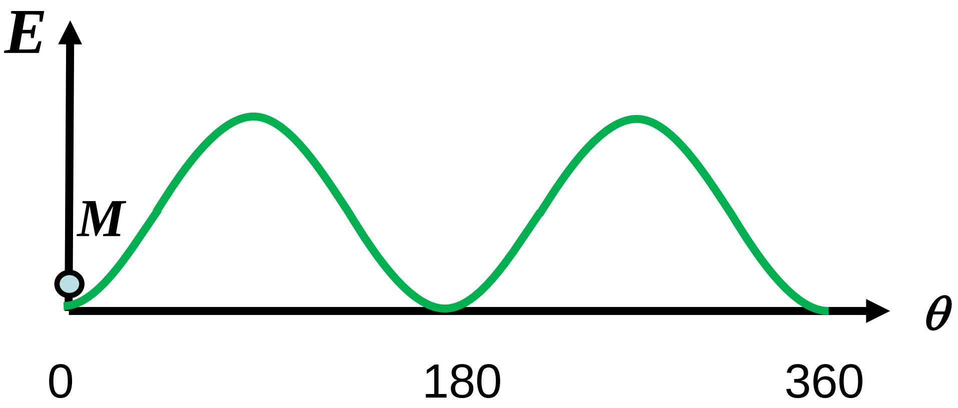
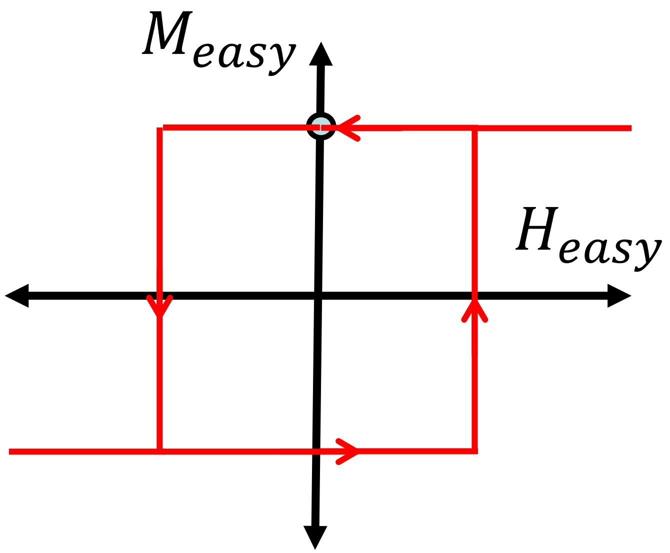




Total energy (Magnetic field along easy axis)





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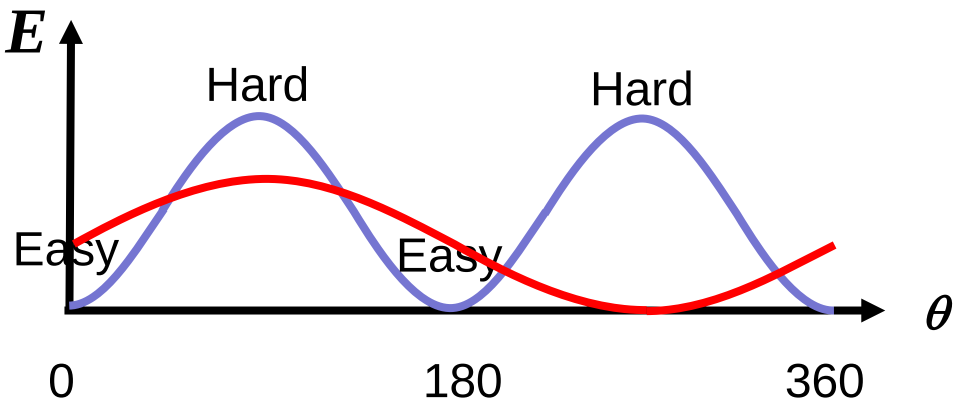
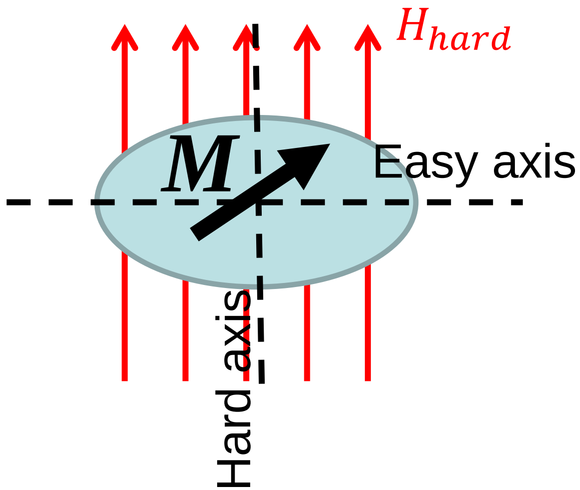
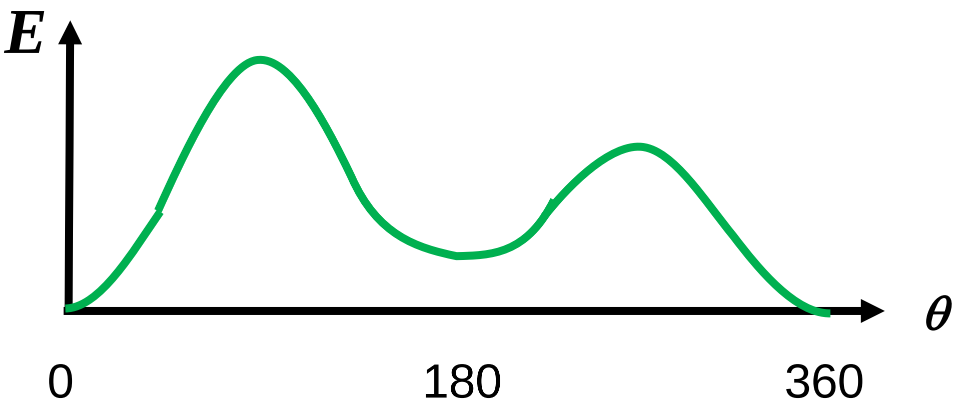




Total energy (Magnetic field along easy axis)



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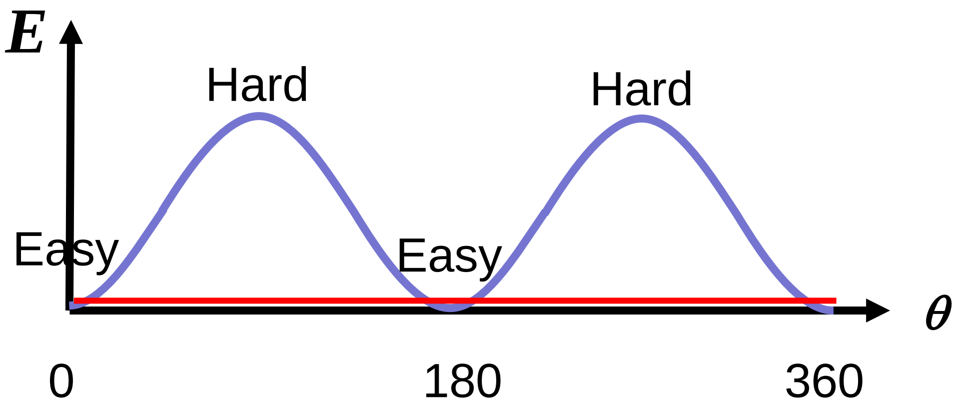
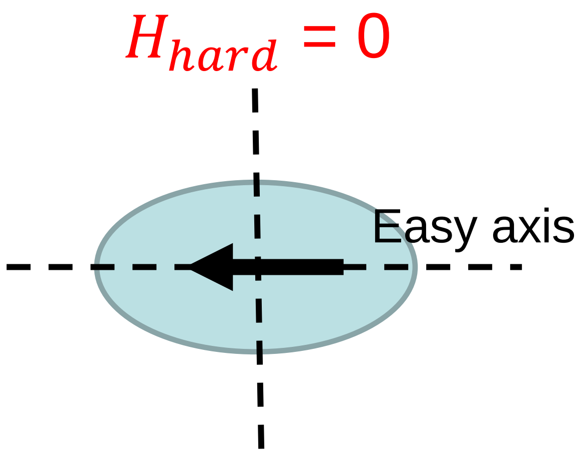
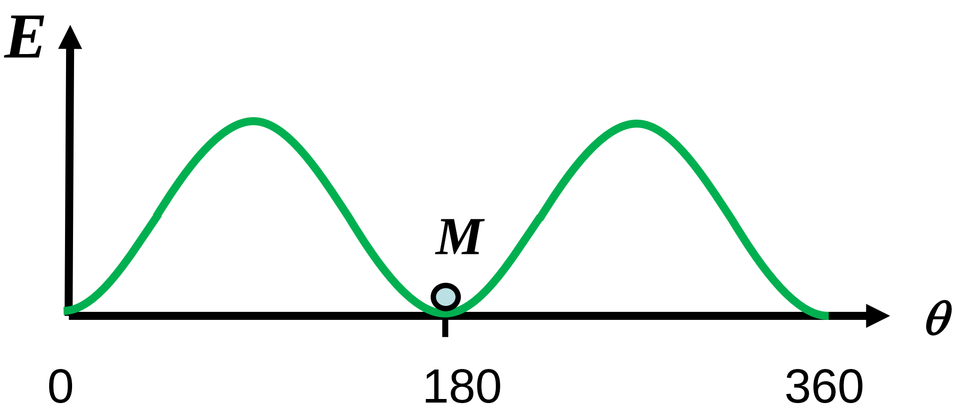
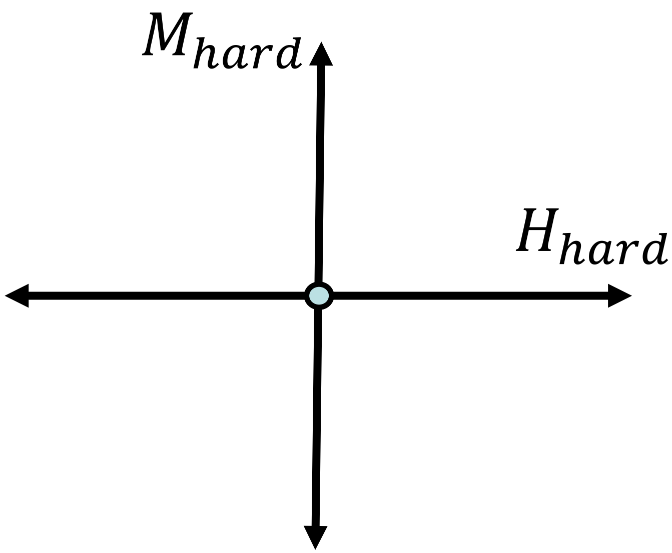




Total energy (Magnetic field along easy axis)





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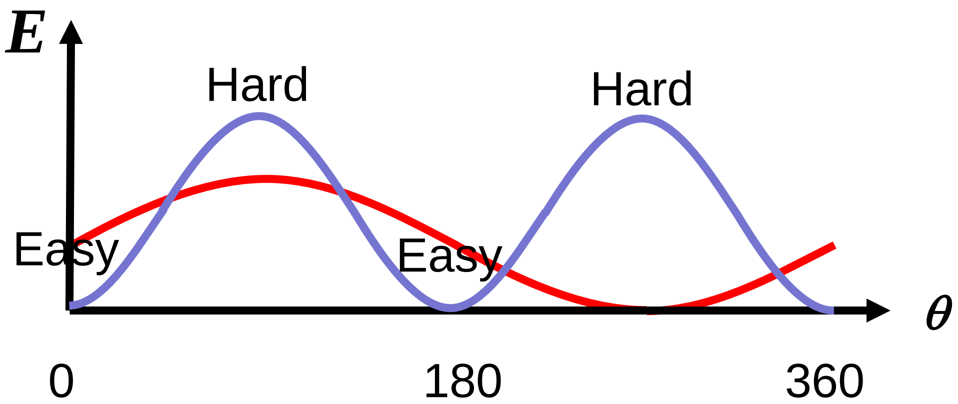
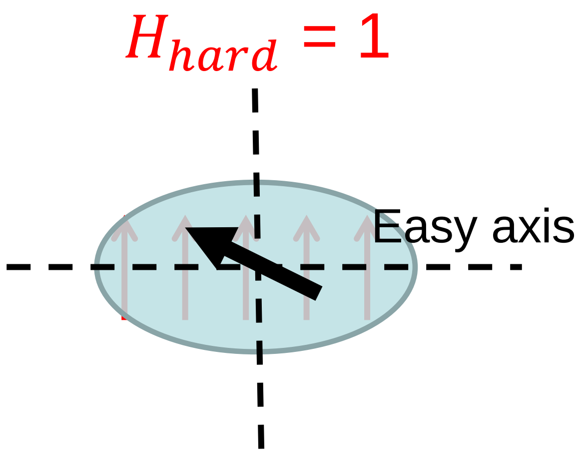
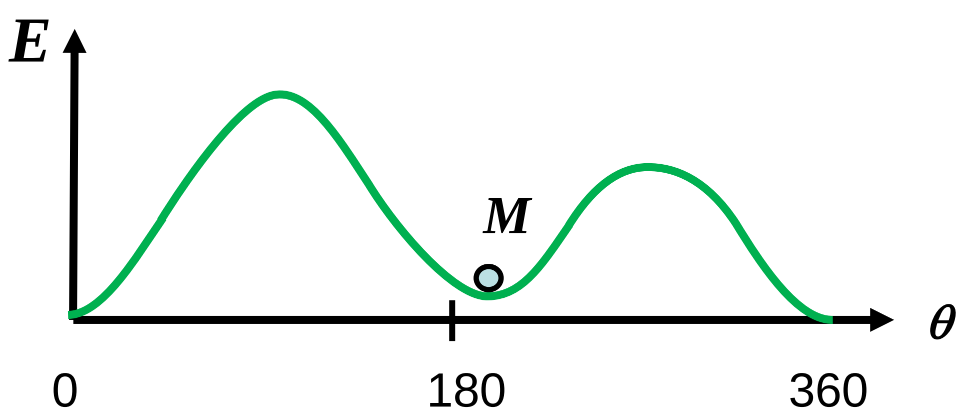
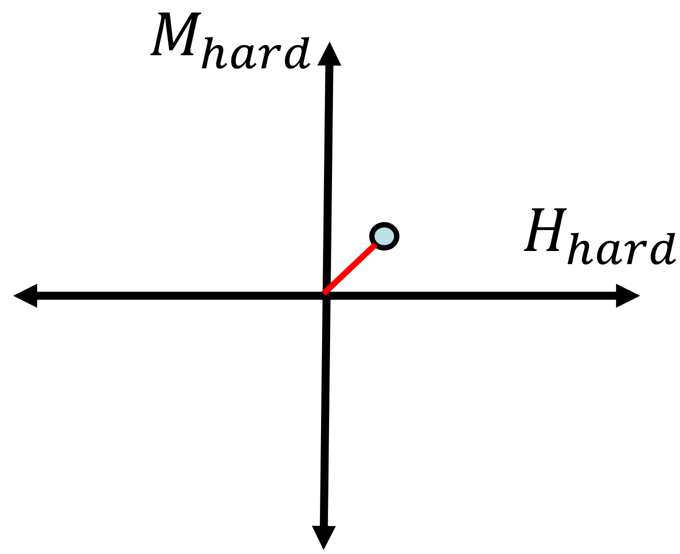




Total energy (Magnetic field along <u>easy axis</u>)





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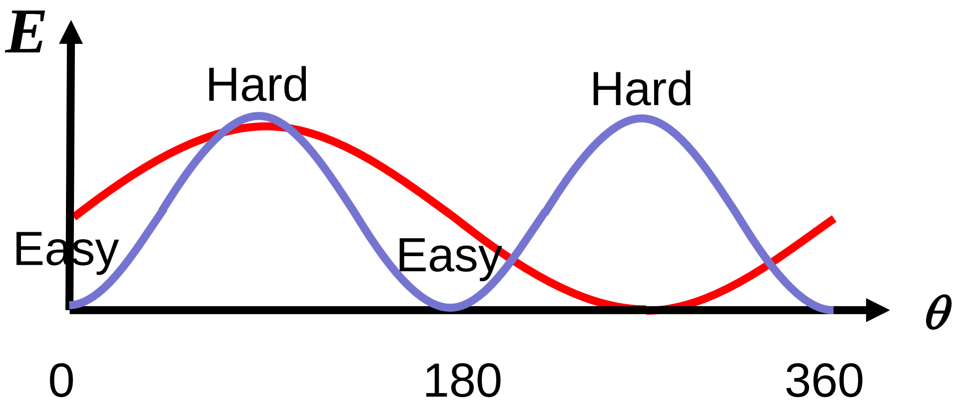
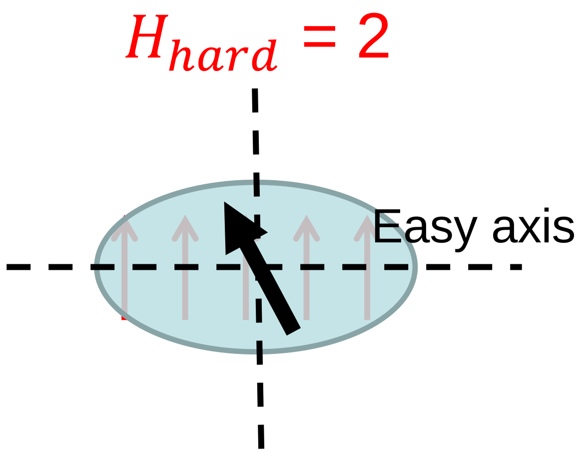
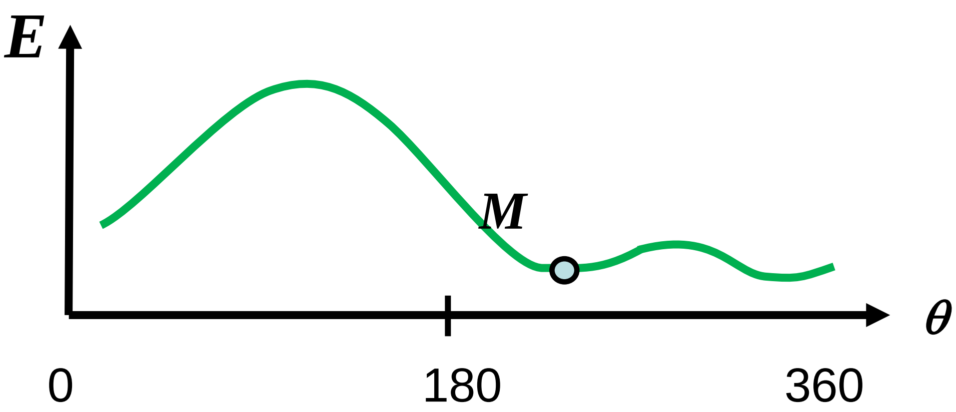
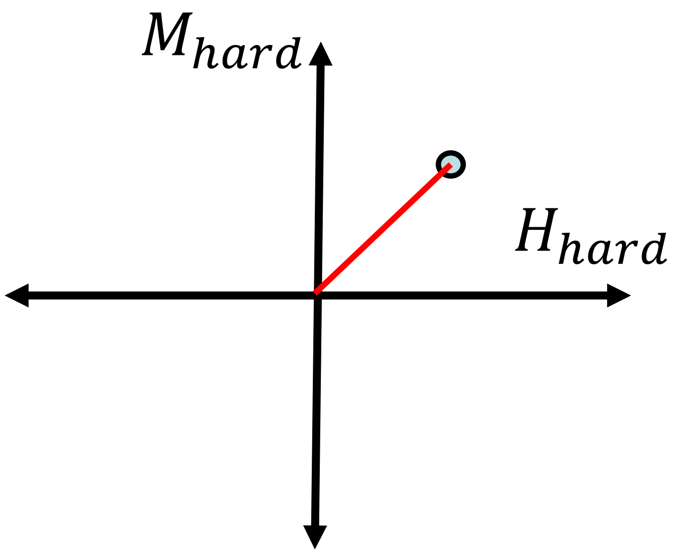




Total energy (Magnetic field along easy axis)





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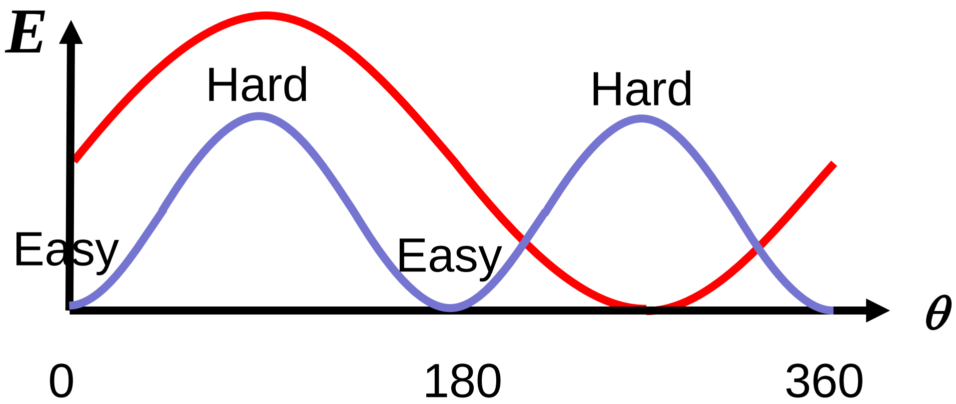
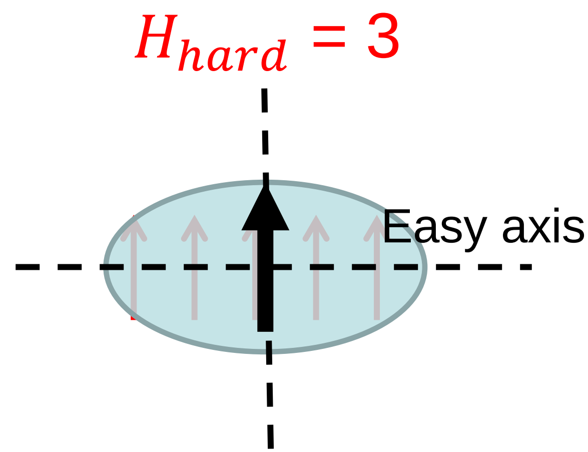
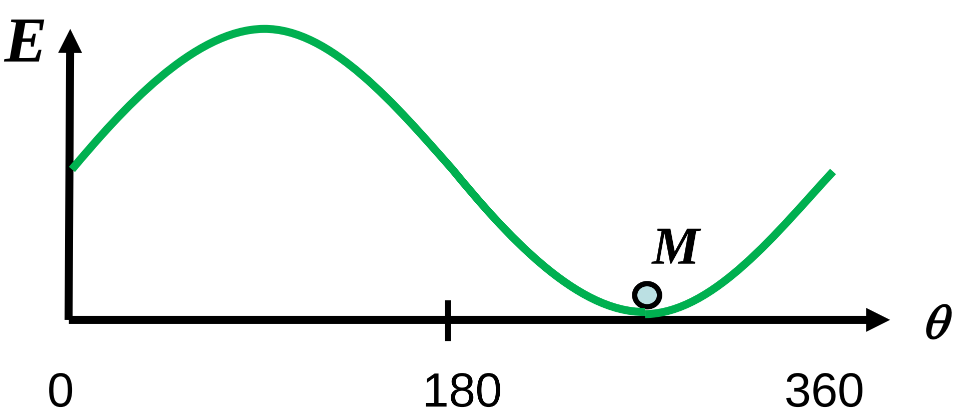
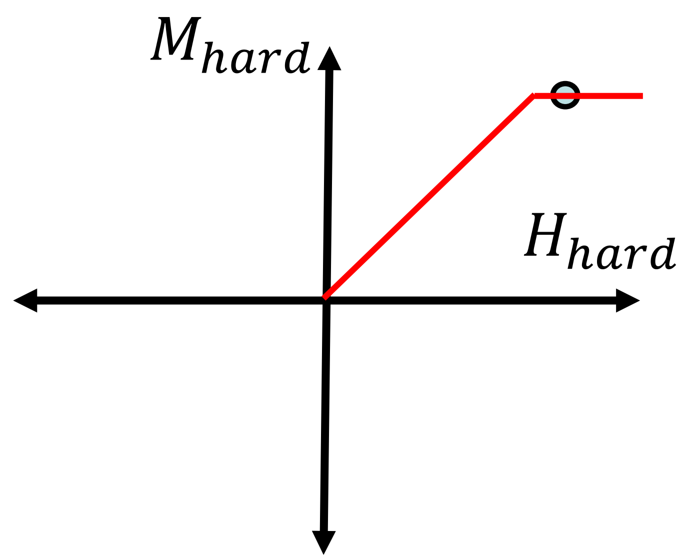




Total energy (Magnetic field along easy axis)





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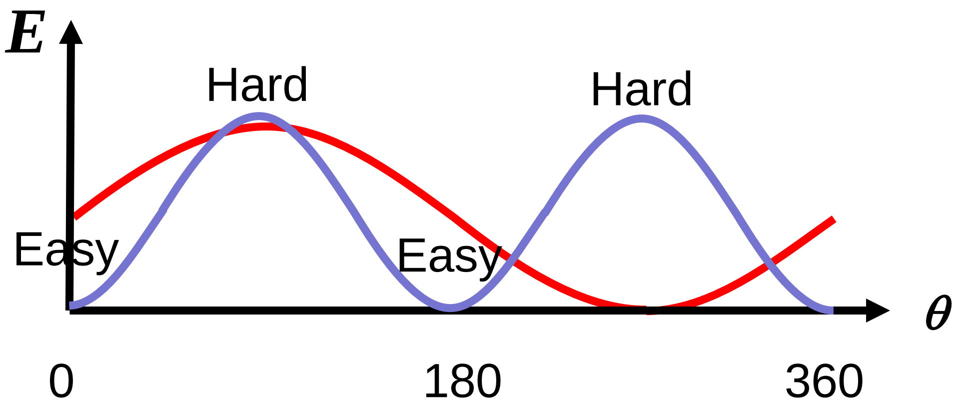
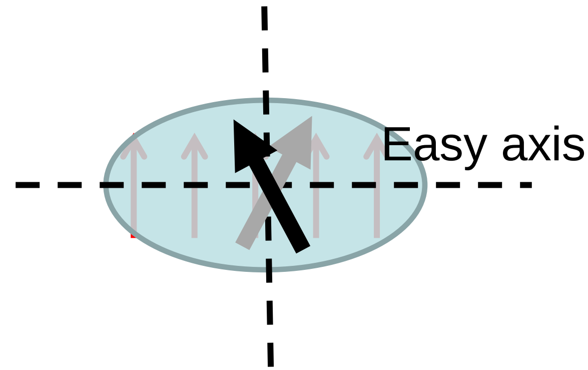
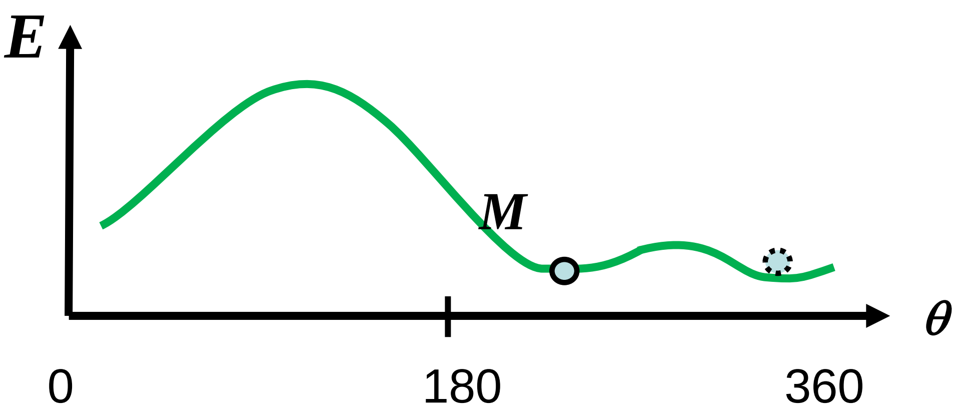
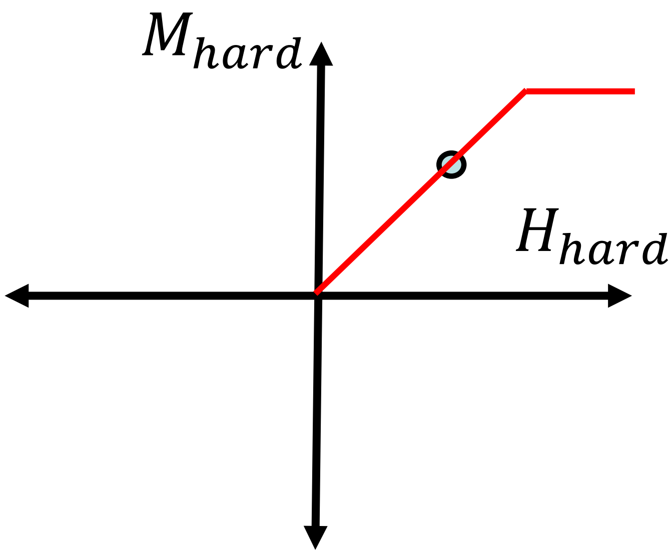


H_{hard} = 2



Total energy (Magnetic field along <u>easy axis</u>)





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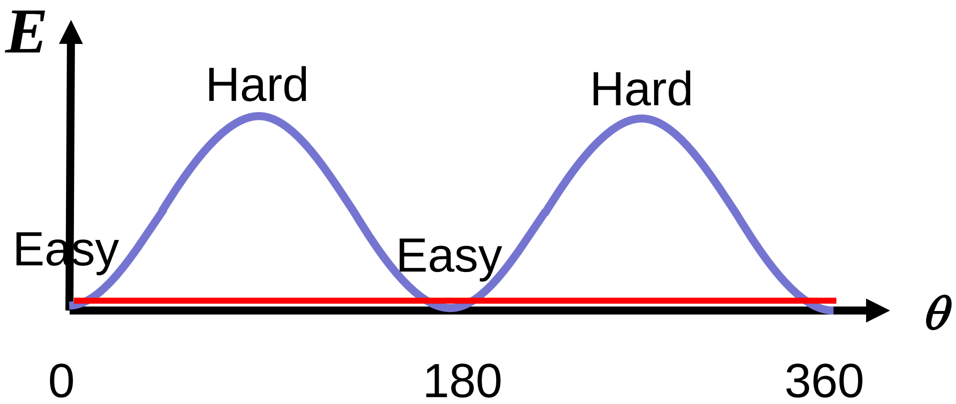
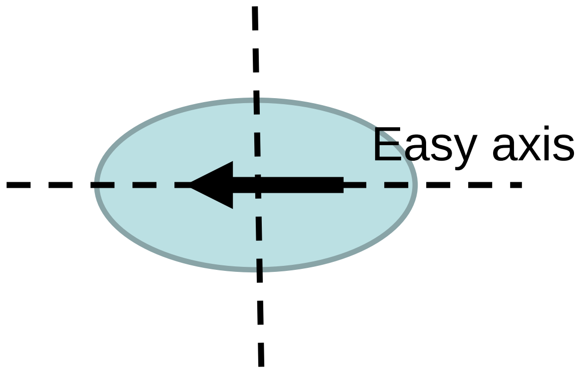
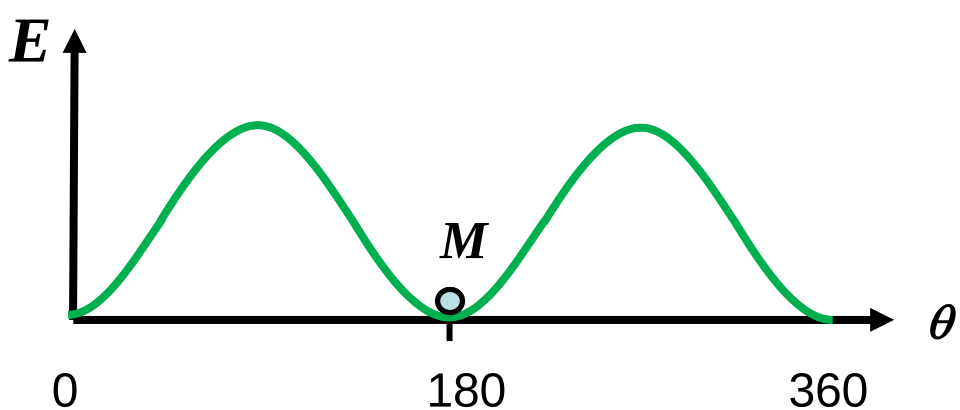
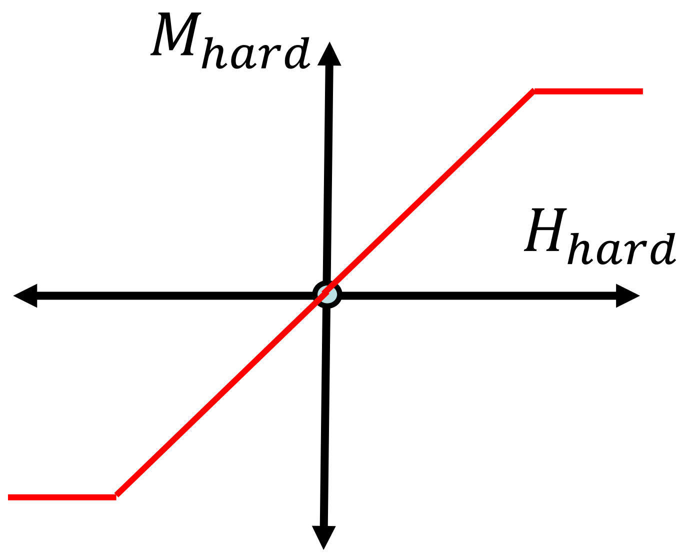


$H_{hard} = 0$



Total energy (Magnetic field along easy axis)





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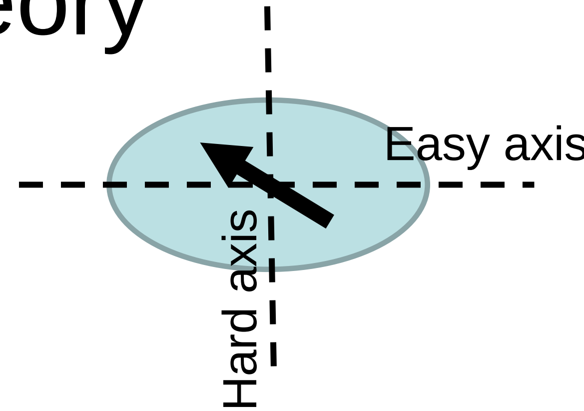
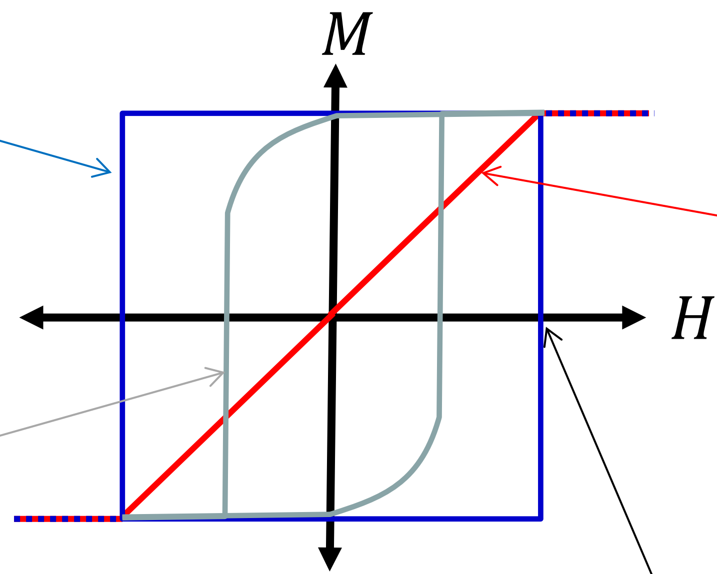


Easy axis loop

- "Hard" magnet
- Permanent magnets
- Data storage



Hard axis loop

- "Soft" magnet
- Transformers/inductors
- Flux guides
- Recording heads

45° loop

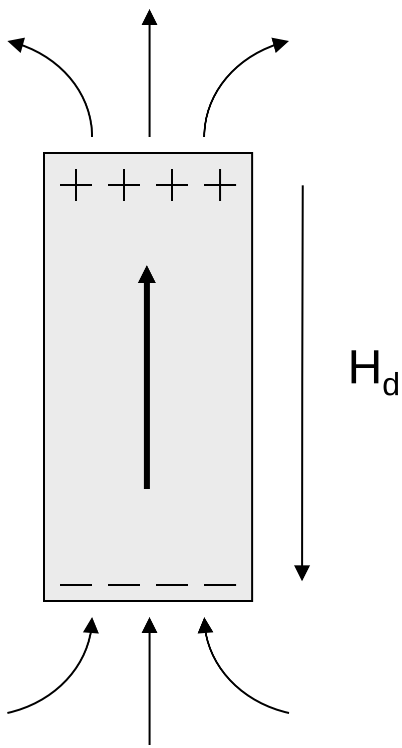
- Lowest switching field
- MRAM

Anisotropy field

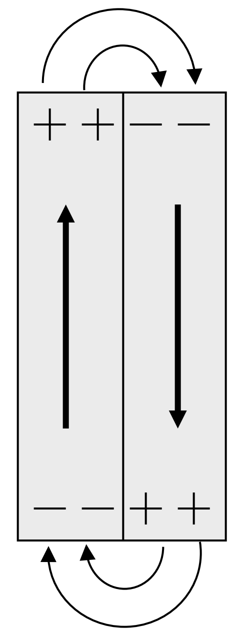
$$H_K = \frac{2K_u}{M_S}$$

Magnetic Domains

Why do Magnetic Domains Form?

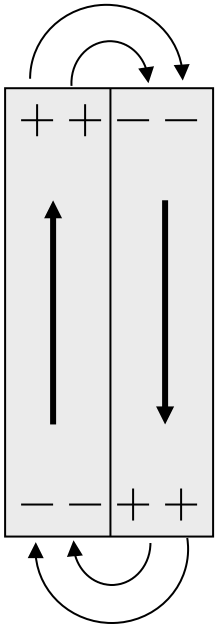


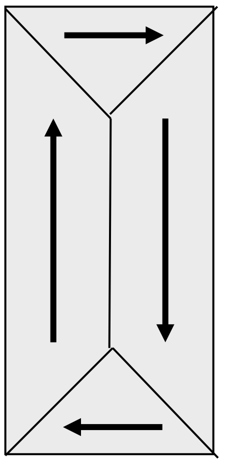
Large demagnetisation field and energy in external field.



Reduced demagnetisation field and external field.

Closure Domains



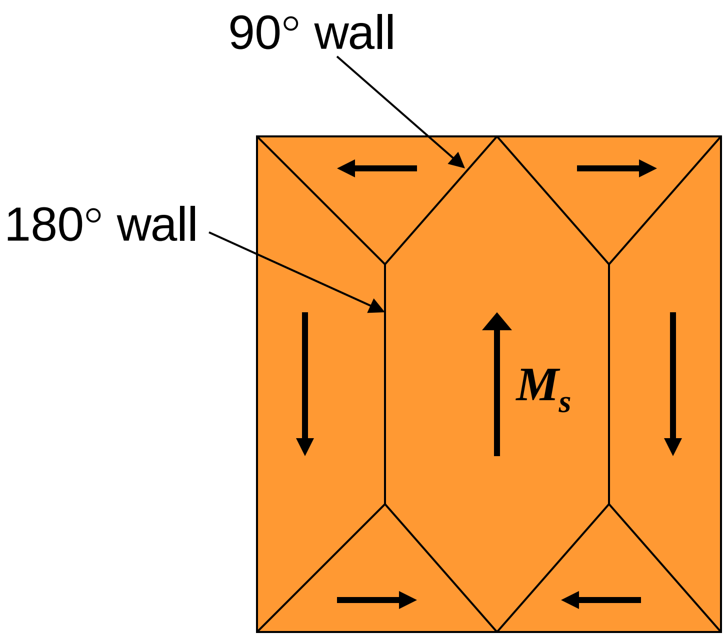


Reduced demagnetisation field and external field.

No demagnetisation field and external field.

Domain Configurations

Domain walls are the boundary between to regions with different magnetization direction.



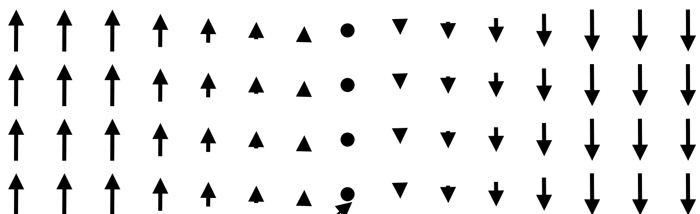
- A ferromagnet with net magnetization less than saturation breaks into domains due to demagnetizing fields.
- Domains form to reduce the magnetostatic energy.
- Within a domain, the magnitude of magnetization is the "spontaneous magnetization", M_s
- The direction of magnetization varies from domain to domain.

Two Types of Domain Walls

Felix Bloch
(1905-1983)

•<u>Bloch wall</u>

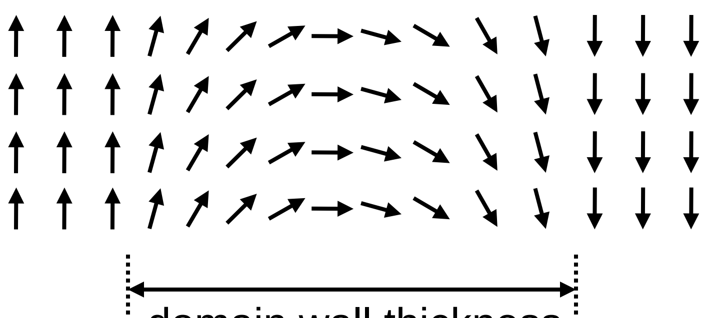
magnetization rotates around axis normal to wall

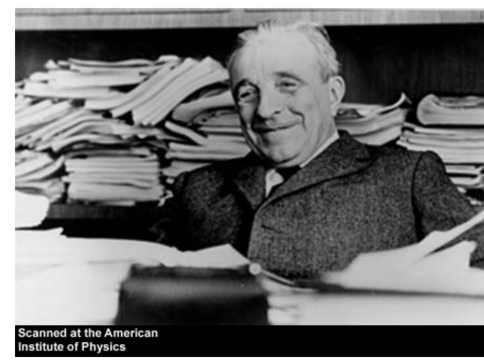


M pointing at you !

$$\mathrm{div}(M) = 0 \quad \Rightarrow \quad \text{nopoles in bulk}$$

•<u>Néel wall</u>

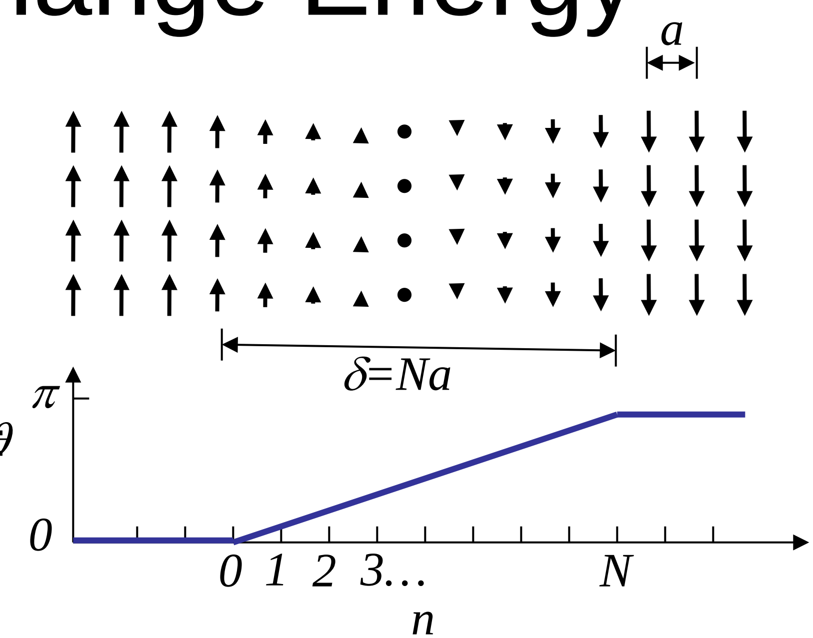




Louis Néel
(1904-2000)

$$\mathrm{div}(M) \neq 0 \quad \Rightarrow \quad \text{distributed poles in bulk}$$

Domain Wall Exchange Energy



- Assume linear transition over N lattice points

$$\theta(n) = \pi \frac{n}{N} \qquad for\ n = 0..N$$

- Angle between adjacent moments is

$$\Delta\theta = \frac{\pi}{N}$$

- Exchange energy between adjacent moments:

$$E_{m_i, m_{i+1}} = -\mu_0 \mathcal{J} m^2 \cos(\Delta\theta) \cong -\mu_0 \mathcal{J} m^2 \left(1 - \frac{(\Delta\theta)^2}{2}\right)$$

- Summing through wall thickness

$$\sum_{n=0..N} E_{m_n, m_{n+1}} = \frac{N \mu_0 \mathcal{J} m^2 (\Delta\theta)^2}{2} + C$$

$$= \frac{\mu_0 \mathcal{J} m^2 \pi^2}{2N} + C$$

- Summing over wall area:

$$E_{exch} = \frac{\text{Area}}{a^2} \frac{\mu_0 \mathcal{J} m^2 \pi^2}{2N} + C$$

- Using $\delta = Na$:

$$\frac{E_{exch}}{\text{Area}} = \frac{\mu_0 \mathcal{J} m^2 \pi^2}{2a} \frac{1}{\delta}$$

- Exchange energy favors thick walls.

Domain Wall Anisotropy Energy

- Assume linear transition over N lattice points

$$\theta(n) = \pi \frac{n}{N} \qquad for\ n = 0..N$$

- Anisotropy energy penalty for each moment:

$$E_{m_i} = K_u \sin^2(\theta) a^3$$

uniaxial anisotropy energy / volume

- Summing through wall thickness

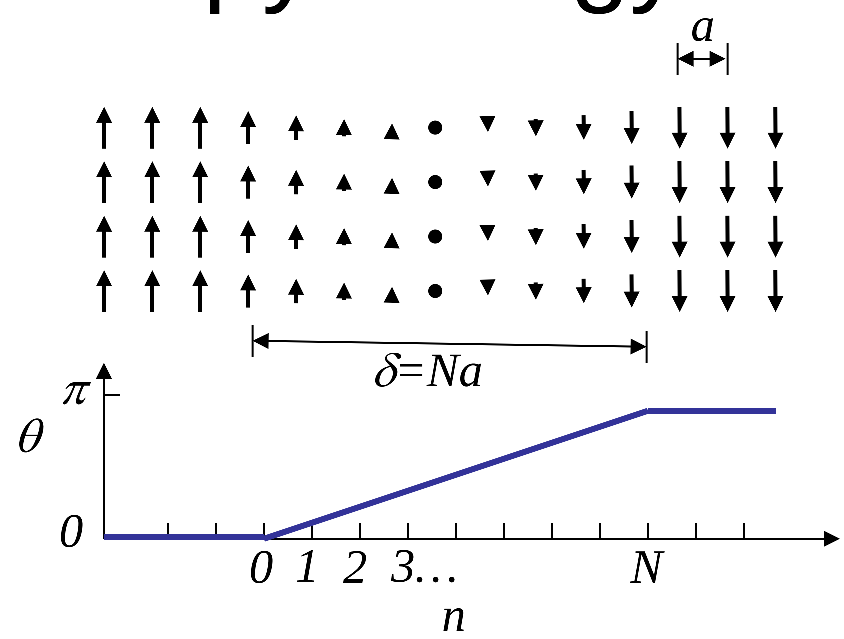
$$\sum_{n=0..N} E_{m_n} \cong \frac{N}{\pi} \int_0^{\pi} K_u \sin^2(\theta) a^3 d\theta = \frac{K_u N a^3}{2}$$

- Summing over wall area:

$$E_{anis} = -\frac{\text{Area}}{a^2} \frac{K_u N a^3}{2}$$

Or,

$$\frac{E_{anis}}{\text{Area}} = \frac{K_u}{2} \delta$$



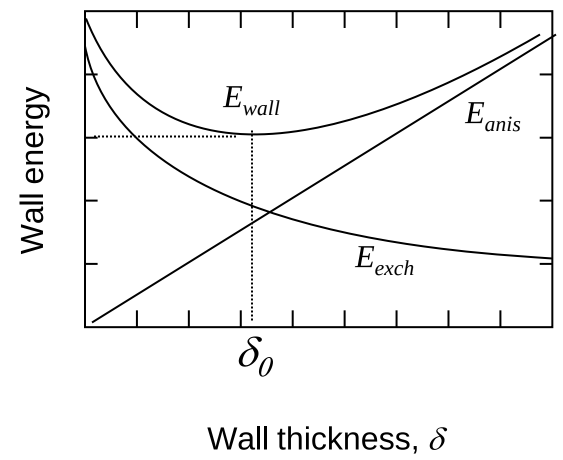
- Anisotropy energy favors thin walls.

Domain Wall Thickness

- Domain wall thickness is determined by a balance between anisotropy and exchange energy:

$$\frac{E_{anis}}{\text{Area}} = \frac{K_u}{2}\delta \qquad \frac{E_{exch}}{\text{Area}} = \frac{\mu_0 \mathcal{J} m^2 \pi^2}{2a}\frac{1}{\delta}$$

- Define exchange stiffness: $A = \dfrac{\mu_0 \mathcal{J} m^2}{a}$



- Total wall energy is:

$$\frac{E_{wall}}{\text{Area}} = \frac{K_u}{2}\delta + \frac{\pi^2 A}{2\delta}$$

- The minimum energy is achieved at:

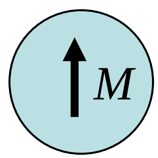
$$\frac{dE_{wall}}{d\delta} = \frac{K_u}{2} - \frac{\pi^2 A}{2\delta^2} = 0 \quad \Rightarrow \quad \delta_0 = \pi\sqrt{\frac{A}{K_u}}$$

- Where the wall energy is:

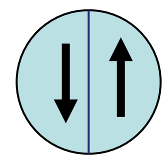
$$E_{wall} = \pi\sqrt{K_u A}\ \text{Area}$$

Single Domain Particles

Single domain



Multi domain



Demag. Energy $\propto r^3$

Less Demag. Energy

Wall Energy $\propto r^2$

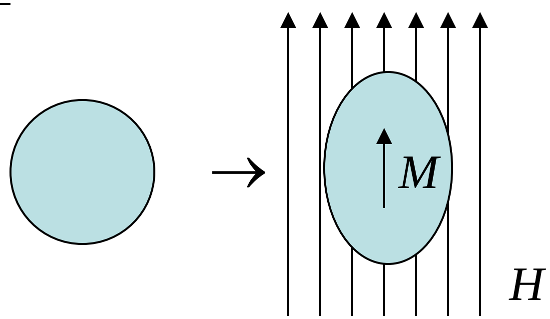
Below a critical size, domain wall can not form.

Magnetostriction

Random crystal orientation:

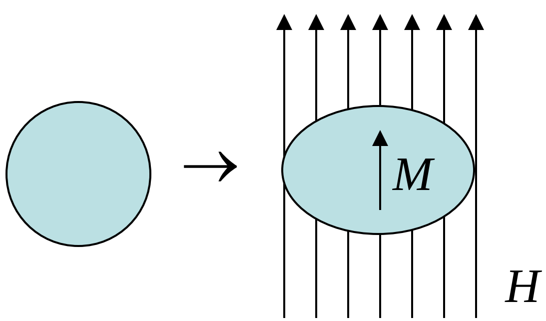
$$\frac{\Delta l}{l} = \frac{3}{2}\lambda_s(\cos^2\theta - \frac{1}{3})$$

$\underline{\lambda_s > 0}$



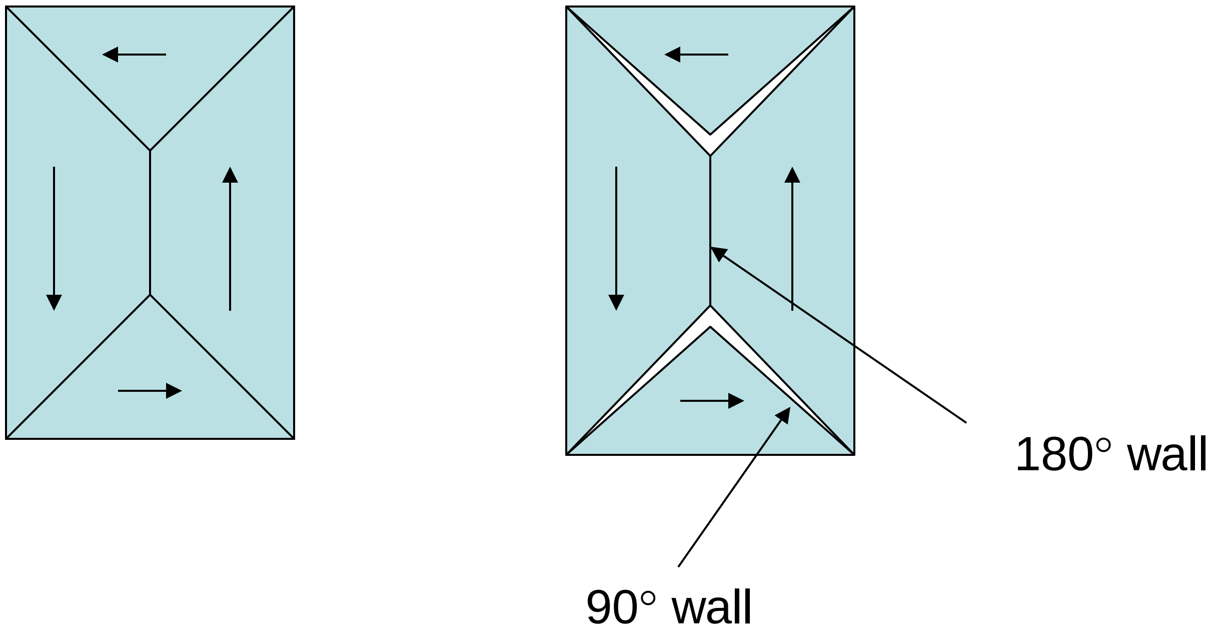
E.g. "Terfenol"
$TbFe_2$: $\lambda_s = 1753 \times 10^{-6}$

$\underline{\lambda_s < 0}$



E.g.
Ni (fcc): $\lambda_s = -34 \times 10^{-6}$
Fe (bcc): $\lambda_s = -7 \times 10^{-6}$

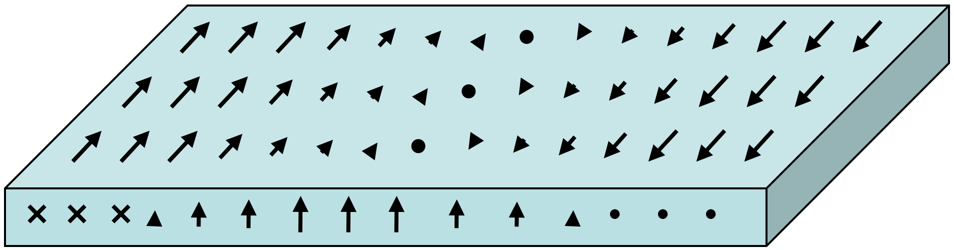
Magnetostrictive Effect on Domain Walls



- Magnetostrictive strain energy favors 180 degree walls.

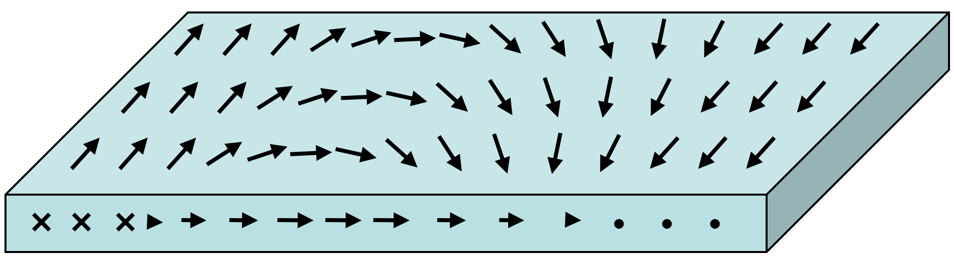
Domain Walls in Thin Films

- Bloch wall

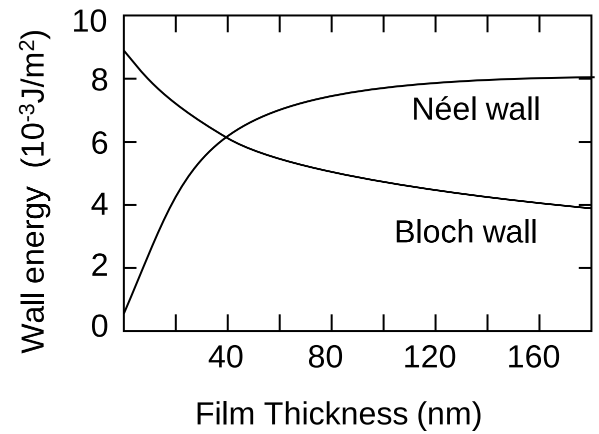


Magnetic poles on surface

- Néel wall



Magnetic poles in bulk

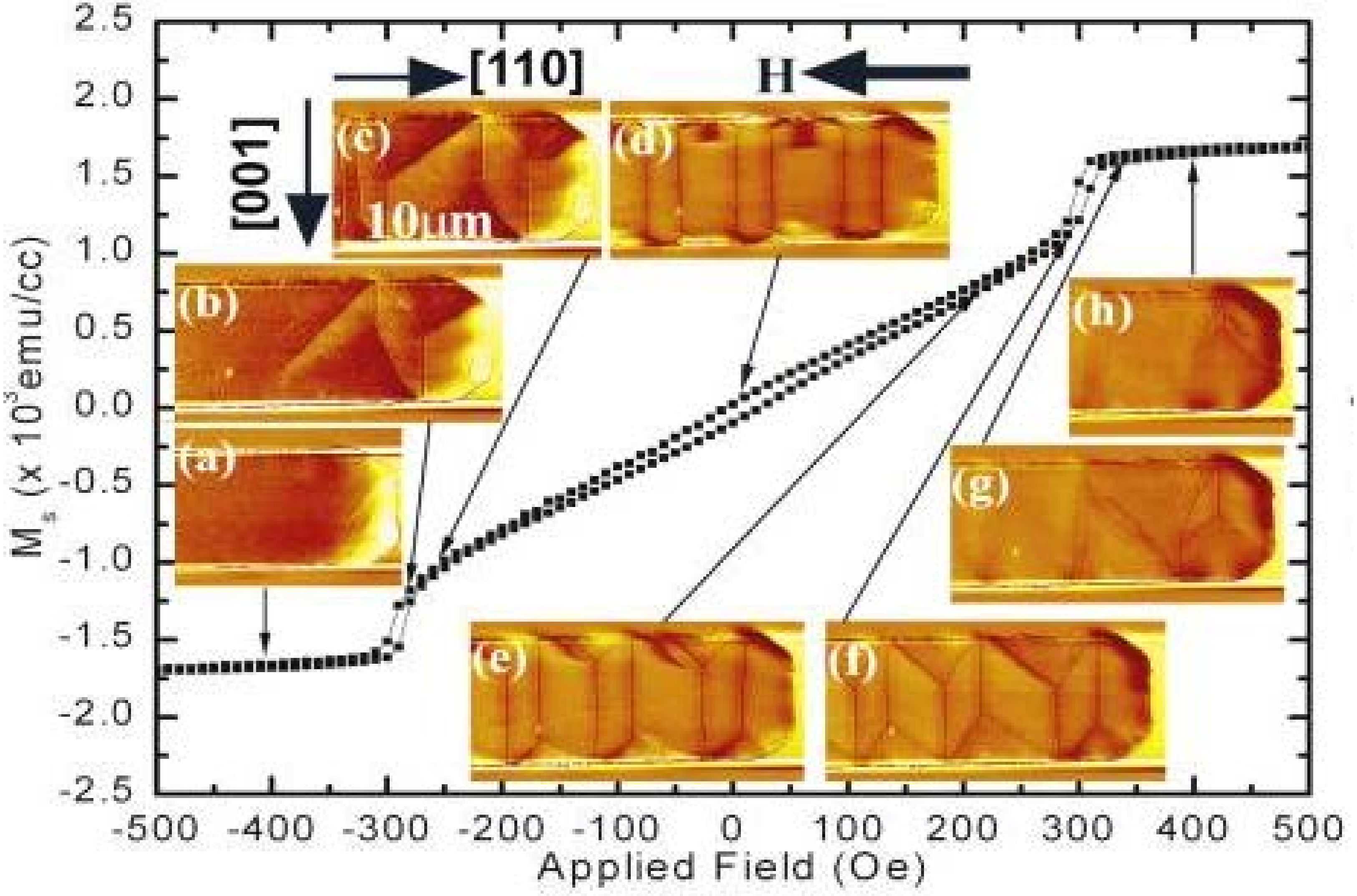


For A = 10^{-11} J/m, B_s = 1 T and K = 100 J/m^3
from O'Handley, Modern Magnetic Materials

Domain Configuration

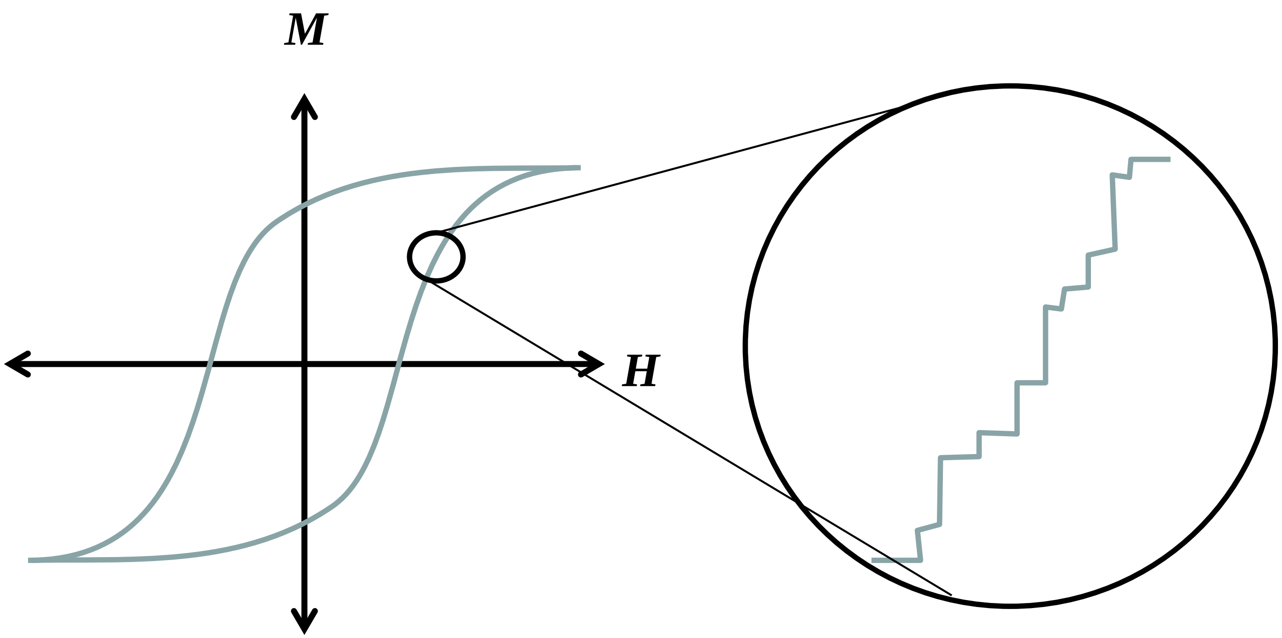
- Balance of energy between:
  - Magnetostatic self energy (demag. field -> try to reduce magnetic poles)
  - Domain wall energy (exchange and anisotropy -> reduce wall area)
  - Strain energy (magnetostriction -> favor 180° walls)
  - Magnetostatic energy in applied field ($E = -m \cdot B$ -> favors alignment with field)

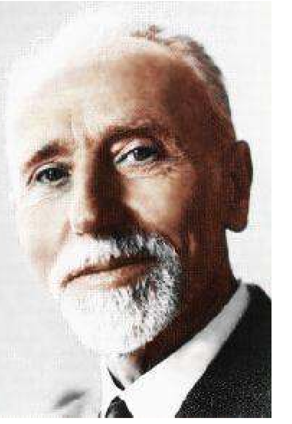
Domains in Fe film



MFM images From: http://physics.unl.edu/~shliou/ResearchActivity/research3.htm

Barkhausen Noise

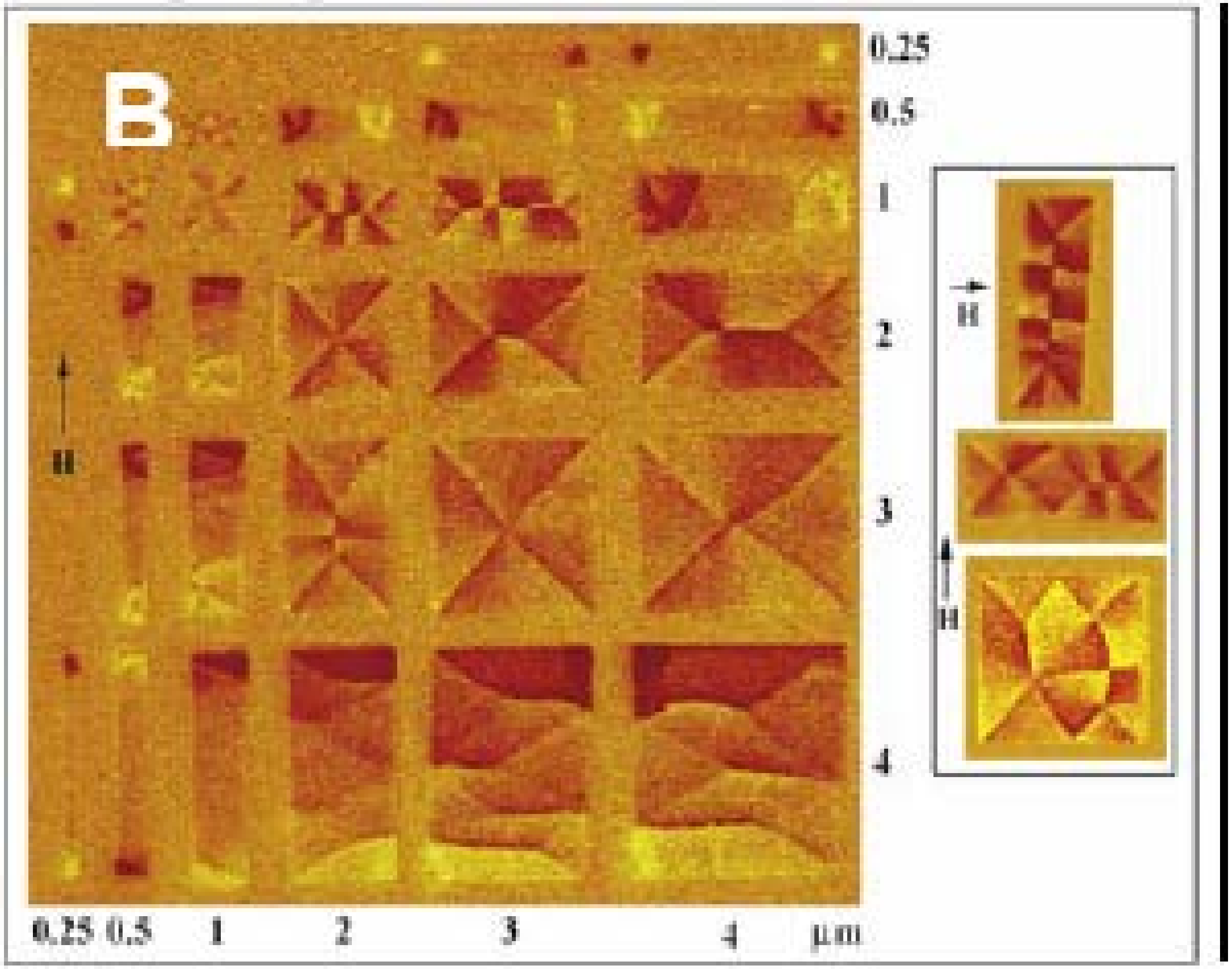




Heinrich Barkhausen
(1881-1956)

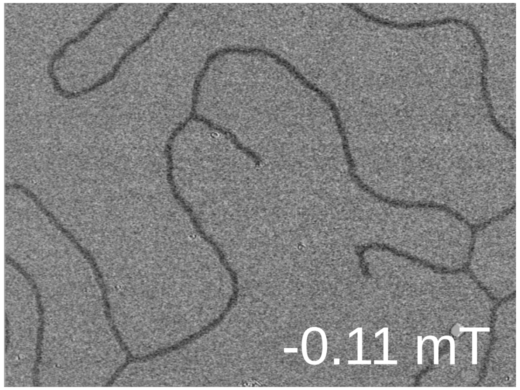
H Barkhausen, Two phenomena uncovered with help of the new amplifiers, Z. Phys, 1919

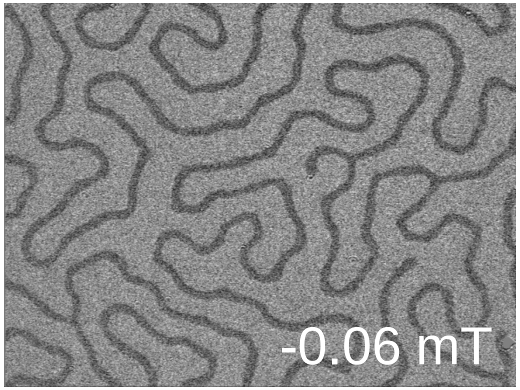
Domains in NiFe patterned film

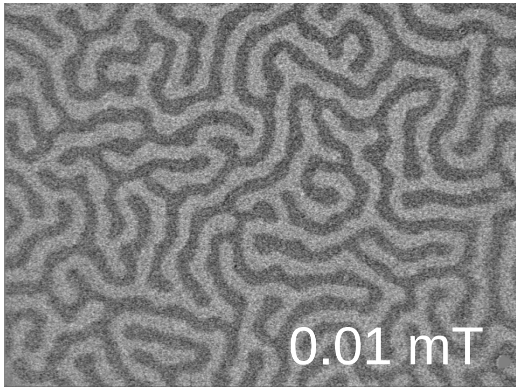


MFM images from
http://www.philsciletters.org/3rd%20week%20June/Visualizing%20objects%20a
t%20nanometer%20scales.htm

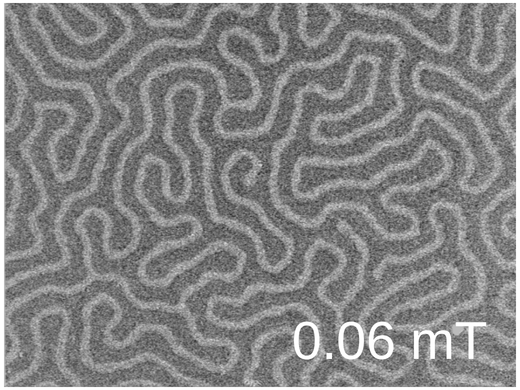
Stripe domains in CoFeB thin film with Perpendicular Anisotropy

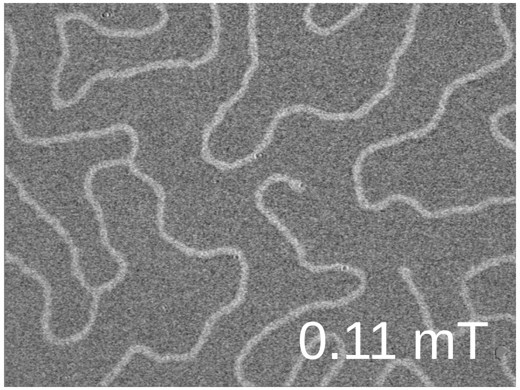






25 μm

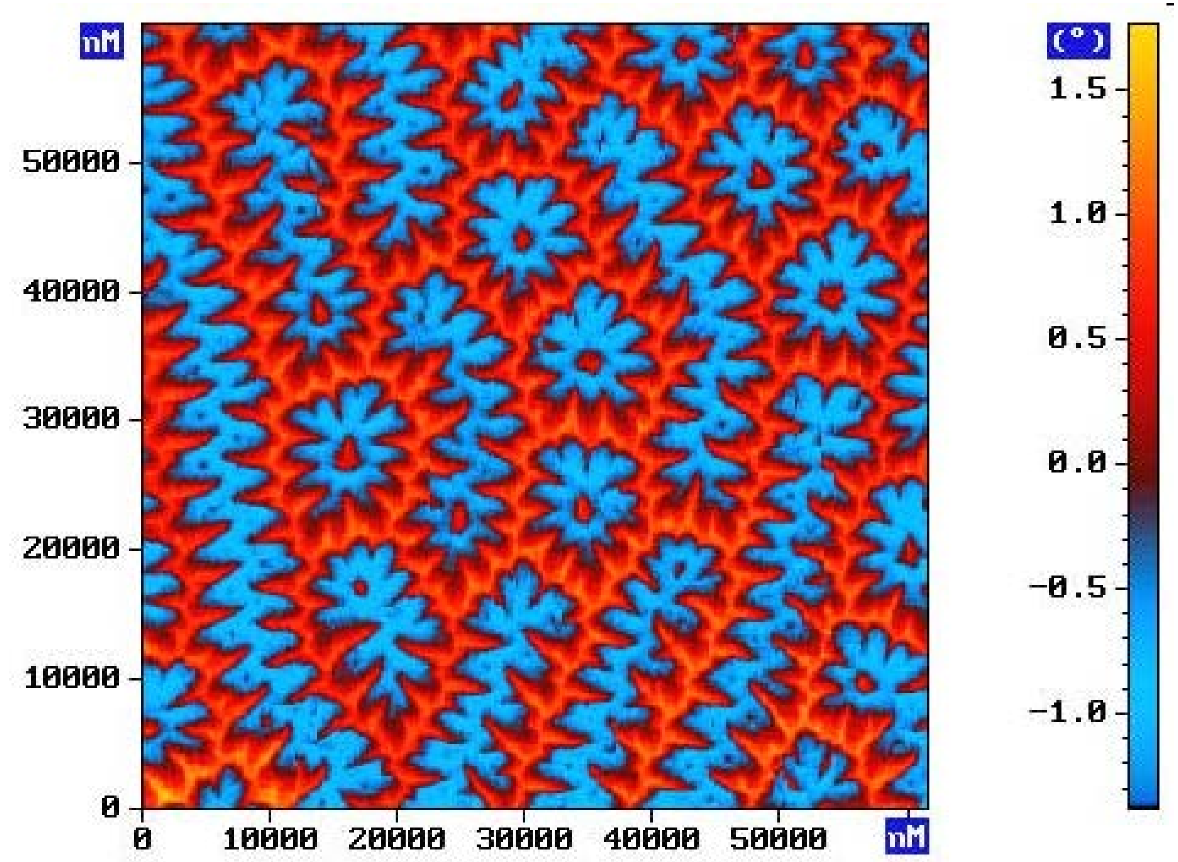




Polar Kerr images.

Flower Domains in YIG

MFM Image



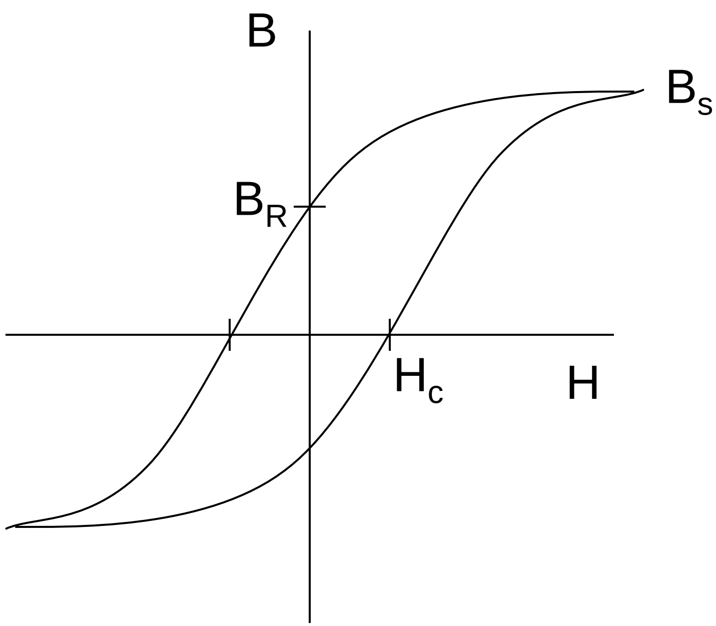
From: http://www.nanoworld.org/russian/s_11.html

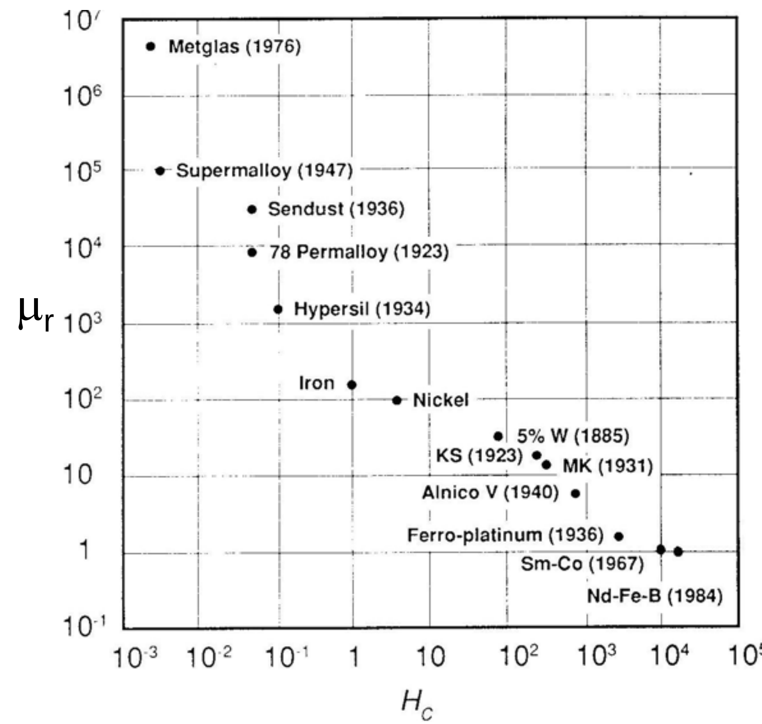
Magnetic Materials

Magnetic materials are typically classified as “hard” or “soft” depending on the intended application.

Soft magnets have low coercivity and high permeability.

Hard magnets have high coercivity and high remanent magnetization.



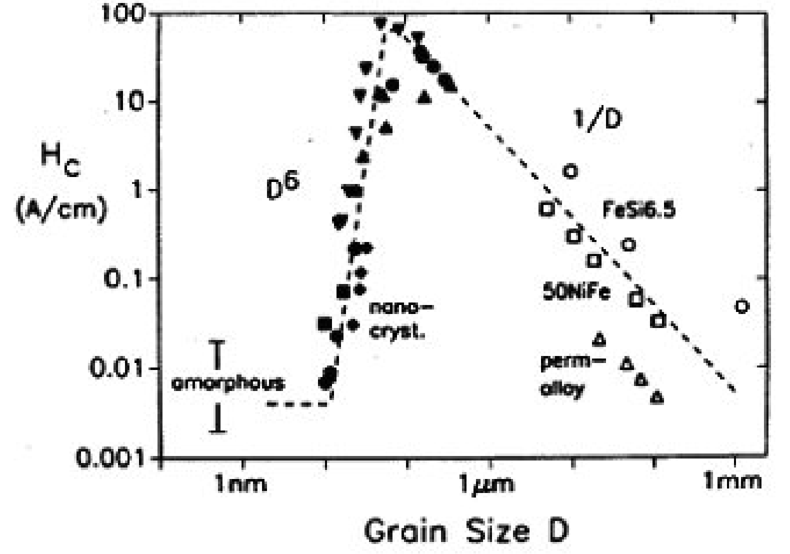


What Determines?

M_s depends primarily on composition.

H_c depends also on :

- grain size/orientation
- crystalline anisotropy
- magnetostriction
- defects



Soft Iron

- Inexpensive, high coercivity material
- μ_r ~ 1000 - 5000
- Used in DC/low frequency applications

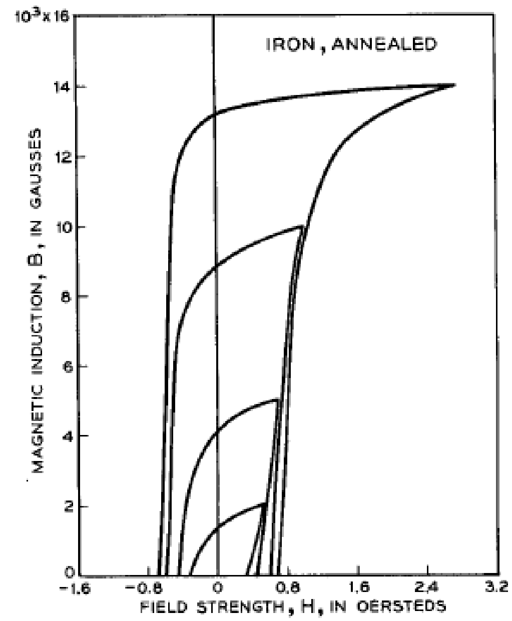
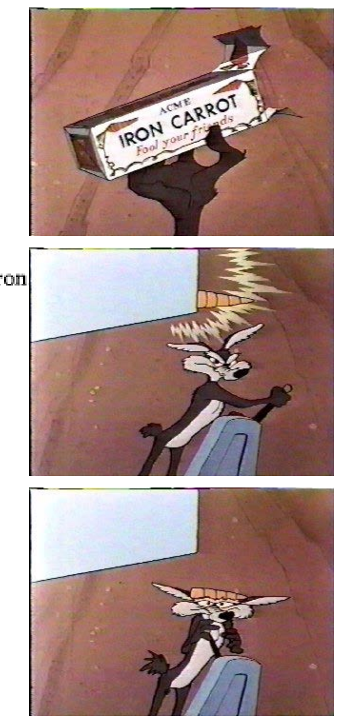
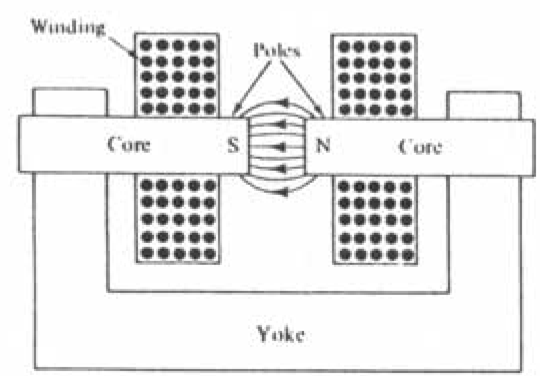


Fig. 3-5. Upper halves of hysteresis loops of ordinary annealed iron

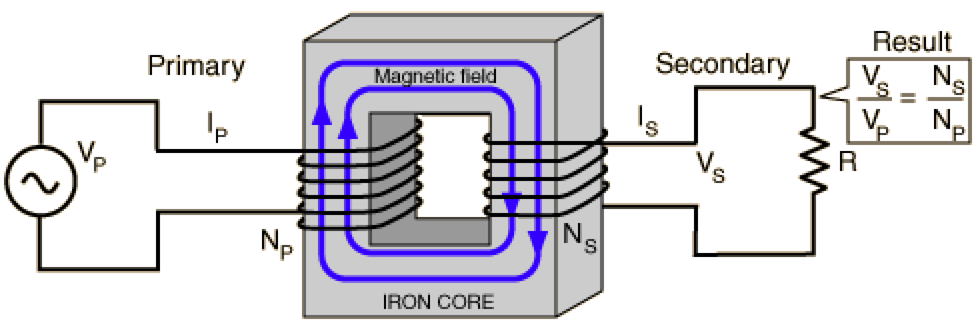




Silicon Steel ("Electrical Steel")

- Most common in power (50/60 Hz) applications
- Adding Si, Al increases resistivity, permeability and reduces coercivity.
- Bad effects: reduces B_s, T_c and increases brittleness

- Higher resistivity and lower coercivity -> less losses
- Higher permeability -> better coupling
- Si limited to 3-4%, material gets too brittle.



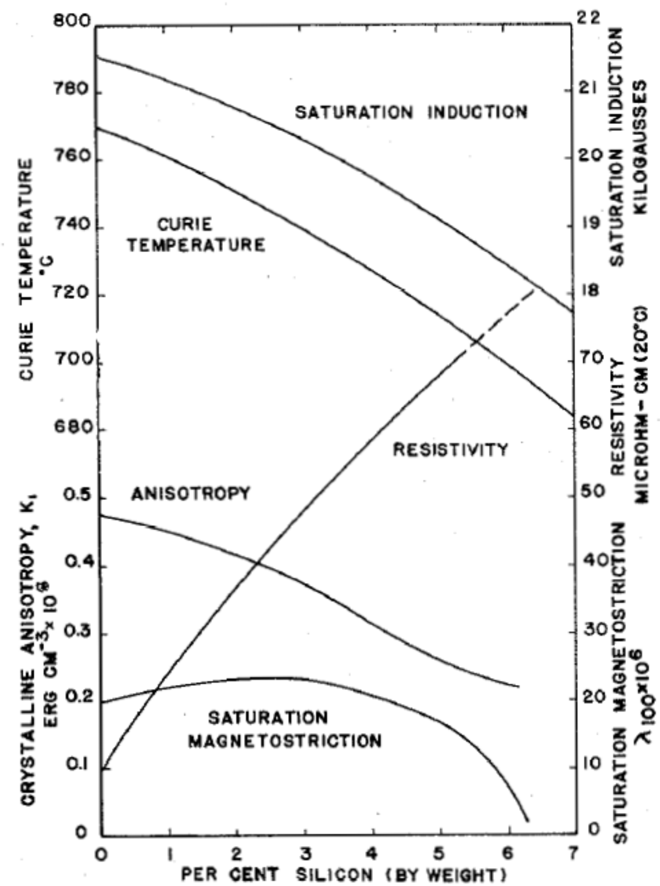


Fig. 1. Variation of important properties of silicon–iron alloys with composition.

Littmann, M.; Iron and silicon-iron alloys,
IEEE Transactions on Magnetics, 7, 48 - 60 (1971)

Fe,Si,Al

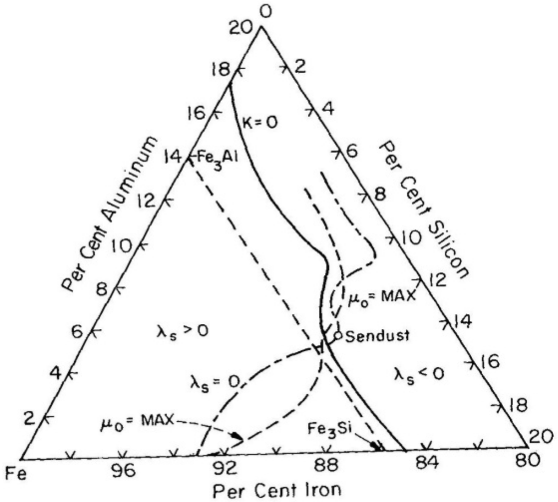


Figure 10.10 Iron-rich corner of ternary Fe-Si-Al diagram (wt%) showing fields of p... tive and negative magnetostriction, the courses of the zero-anisotropy line and maxim... permeability line. The Sendust composition is defined by the intersection of the λ_s = and $K = 0$ lines (Bozorth, IEEE Press, copyright 1994).

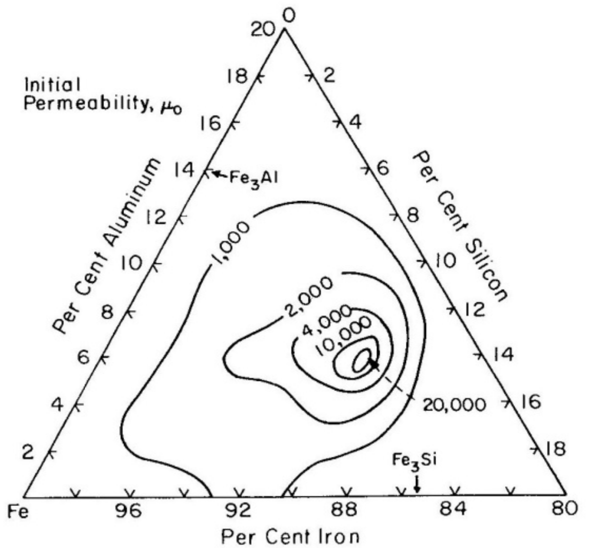


Figure 10.11 Contours of initial permeability on same Fe-Si-Al ternary diagram as ... Figure 10.10 (Bozorth, IEEE Press, copyright 1994).

Cold-rolled Grain-Oriented Steel (CRGO)

•Reduces hysteresis further by orienting grains:

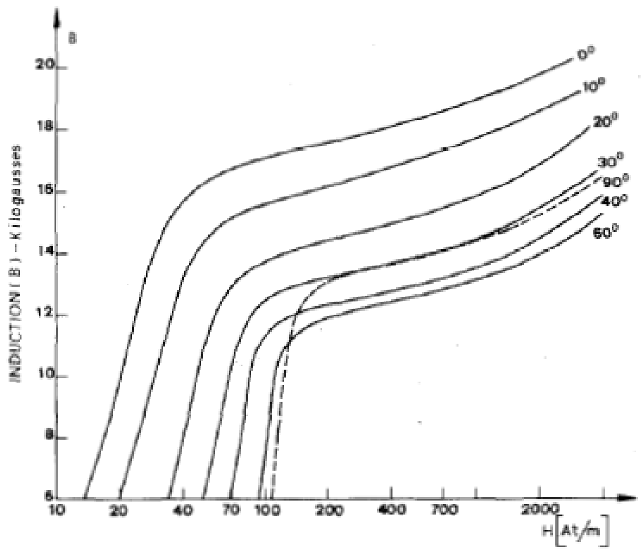
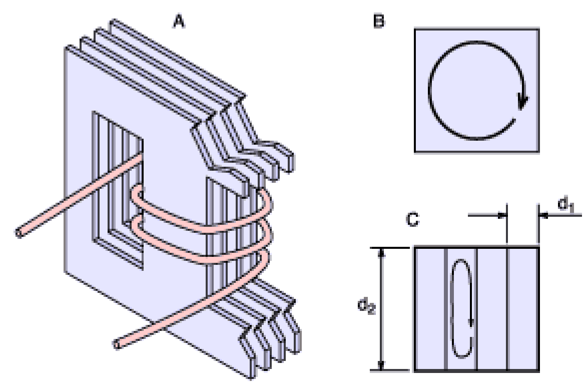


Fig. 2. AC excitation curve, induction at various angles to rolling direction.

*

	Losses (W/kg)
Commercial iron	5-10
Si-Fe hot rolled	1-3
Si-Fe CRGO	0.3-0.6

Lamination reduces eddy currents:



*Di Napoli, A.; Paggi, R.; A model of anisotropic grain-oriented steel, IEEE Transactions on Magnetics 19, 1557, (1983)

Ni-Fe Alloys (Permalloys)

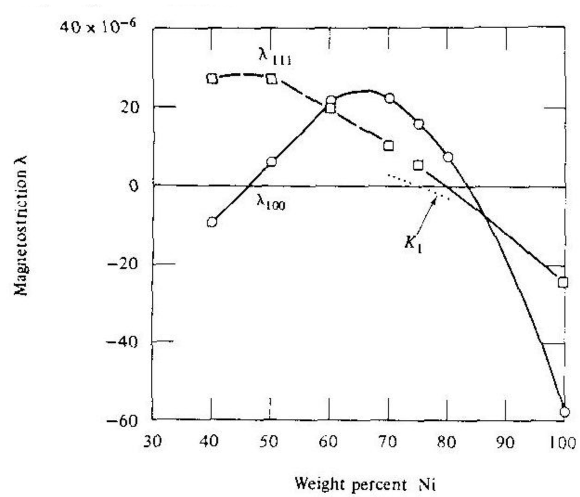


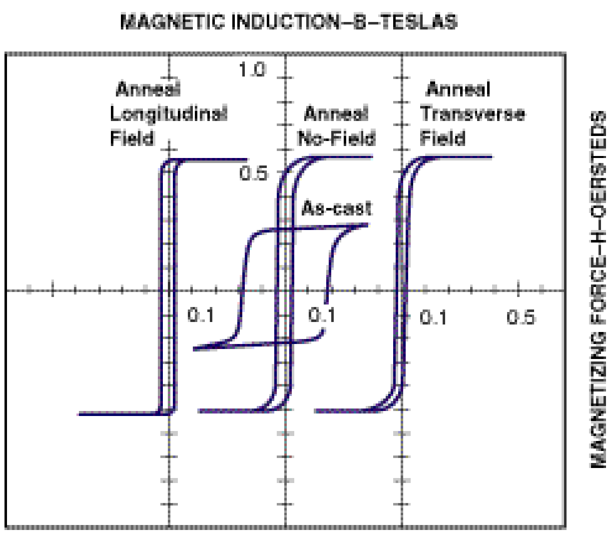
Fig. 12.12 Dependence of magnetostriction coefficients λ_{100} and λ_{111} with nickel conte... r iron–nickel alloys.

Zero magnetostriction at 80%

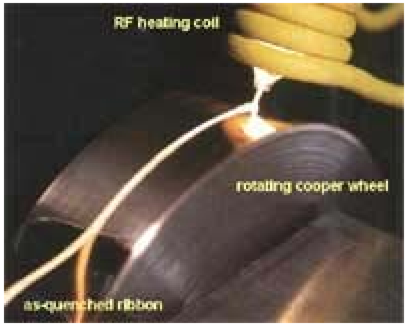
Zero anisotropy at 78%

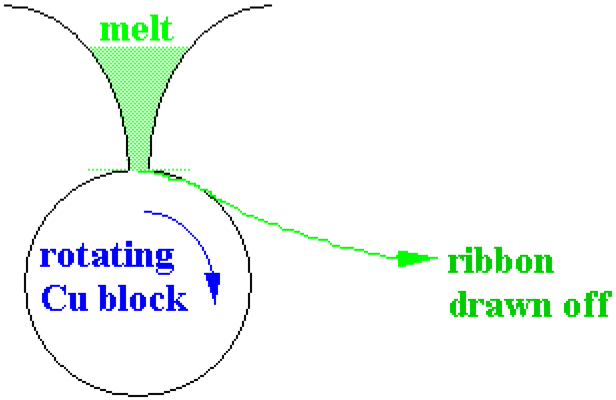
Amorphous Alloys, "Metglas" (Metallic Glasses)

- No crystalline anisotropy
- Ultra-low coercivity and super-high permeability
- Available in thin ribbons only:



- Rapid quenching: melt spinning





Some Soft Magnetic Materials

Material	Composition	Relative Permeability		H_c (kA/m)	B_s (T)
		μ_i	μ_{max}		
Iron	100% Fe	150	5000	80	2.15
Silicon-iron (non-oriented)	96% Fe 4% Si	500	7000	40	1.97
Silicon-iron (grain oriented)	97% Fe 3% Si	1500	40,000	8	2.0
78 Permalloy	78% Ni 22% Fe	8000	100,000	4	1.08
Hipernik	50% Ni 50% Fe	4000	70,000	4	1.60
Supermalloy	79% Ni, 16% Fe 5% Mo	100,000	1,000,000	0.16	0.79
Mumetal	77% Ni, 16% Fe 5% Cu, 2% Cr	20,000	100,000	4	0.65
Permendur	50% Fe 50% Co	800	5000	160	2.45
Hiperco	64% Fe, 35%Co 0.5% Cr	650	10,000	80	2.42
Supermendur	49% Fe, 49%Co 2% V	-	60,000	16	2.40
Metglass (amorphous)	1.6%Ni,4.4%Fe, 8.6%Si,82%Co, 3%B	-	1,000,000	0.01	0.57

Ferrites

Ceramic materials of type $MO{\cdot}Fe_2O_3$ where M = transition metal

Magnetic properties are not as good as metals.
But, high resistivity makes them useful for high frequency applications.

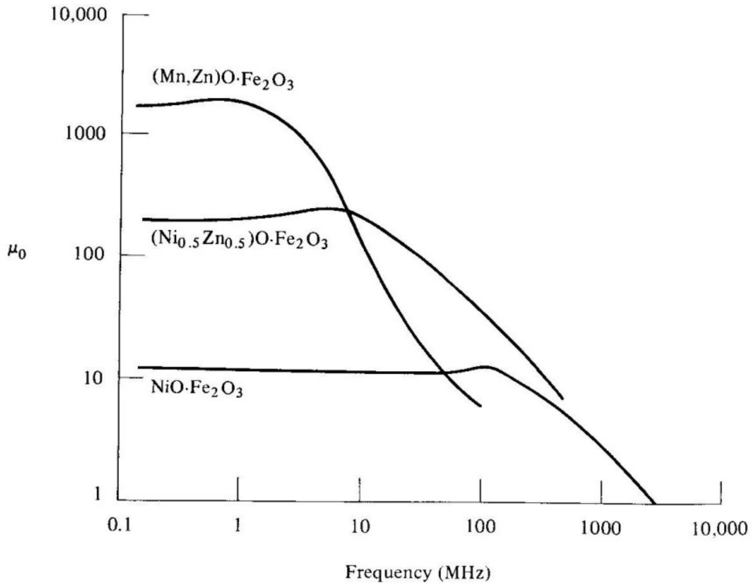


Fig. 13.42 Variation of initial permeability μ_0 with frequency for three ferrites.

Skin depth:

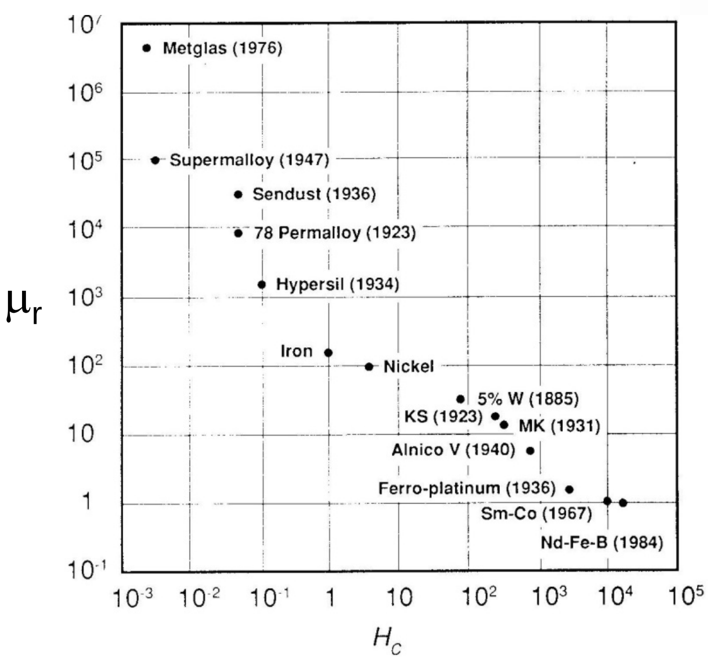
$$\delta = \frac{1}{\sqrt{\mu\sigma\pi f}}$$

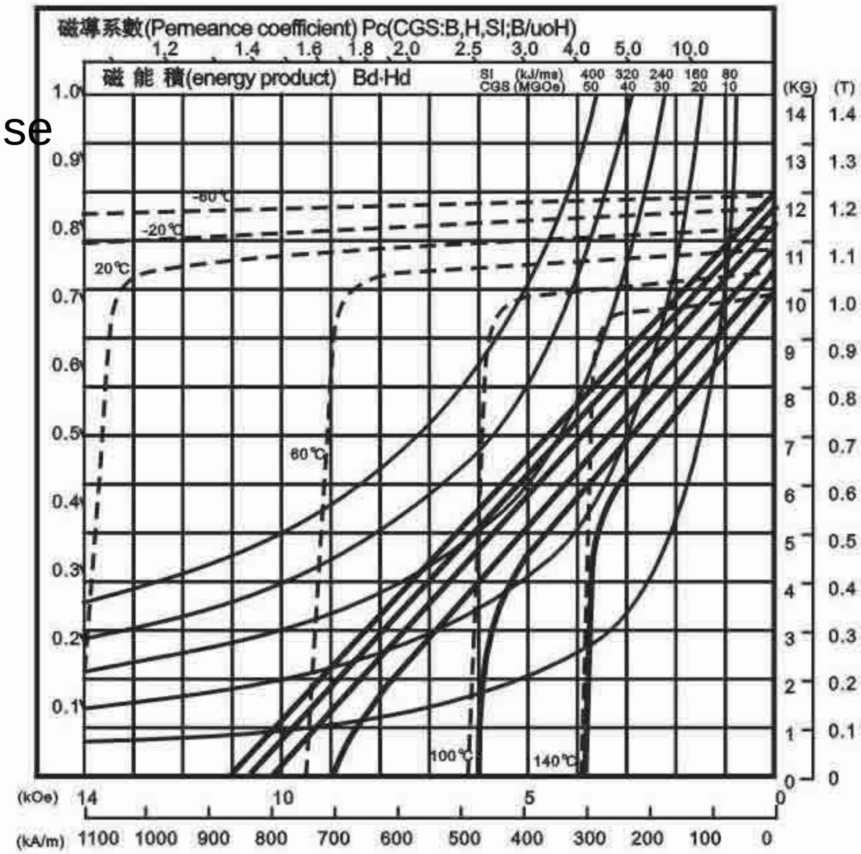
= 8.6 mm for Cu at 60 Hz

= 3 mm for Si-Fe at 60 Hz.

Hard Magnetic Materials

- Permanent magnet materials.
- Manufacturers typically provide data in the form of second quadrant B-H response or “demagnetisation curves”
- Often B and $\mu_0 M$ are plotted on same axis and often in CGS units.
- These days often in Chinese.

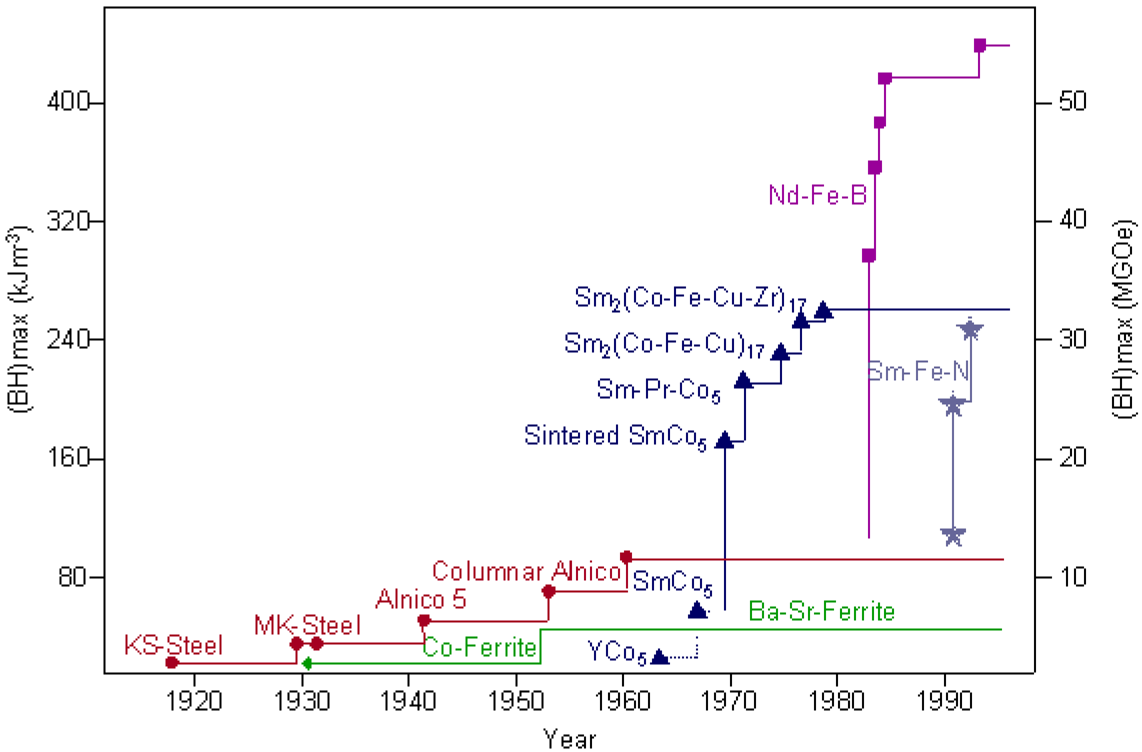




N-30

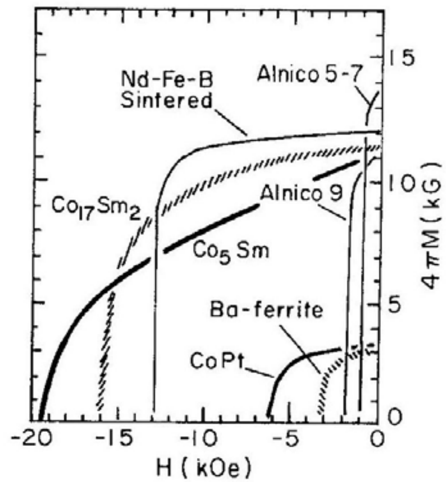
Demagnetization curves of sintered magnet N30 at different temperatures

Advances in Permanent Magnet Materials



http://www.aacg.bham.ac.uk/magnetic_materials/history.htm

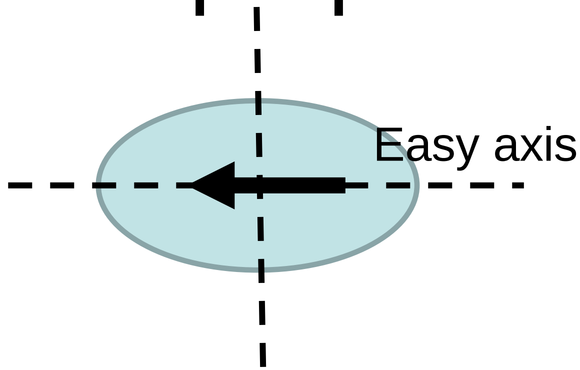
Common Permanent Magnet Materials



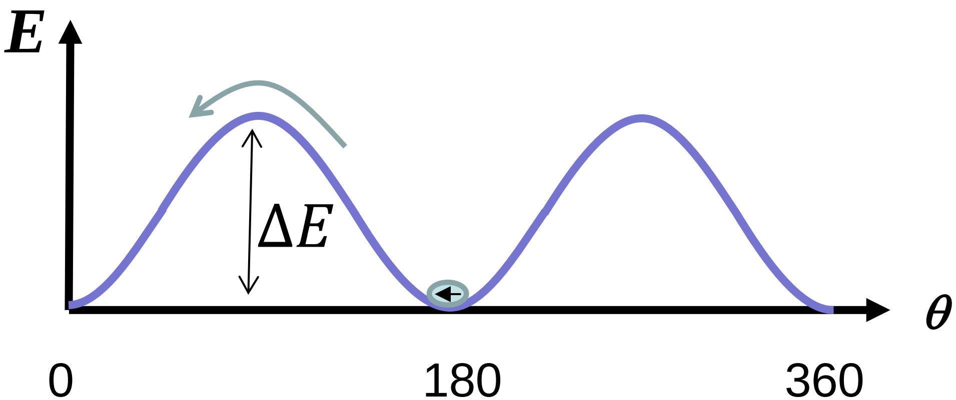
Material	B_r (T)	H_c (kA/m)	$(BH)_{max}$ (kJ/m^3)	T_c of B_r (%/°C)	T_{max} (°C)	T_{curie} (°C)	Cost (relative)	Notes
SmCo	1.05	730	206	-0.04	300	750	Highest (100)	Hard, brittle
NdFeB	1.28	980	320	-0.12	150	310	High (50)	Corrodes easily
Alnico	1.25	50	45	-0.02	540	860	Medium (30)	
Ferrite	0.39	250	28	-0.20	300	460	Low (5)	
Bonded	0.25	160	11	-0.19	100	-	Low (1)	Injection molded

Thermal Decay of Magnetization

Superparamagnetism



For H_a=0:



$$\Delta E \quad = K_u V$$

$$M(t) = M_0 e^{-t/\tau}$$

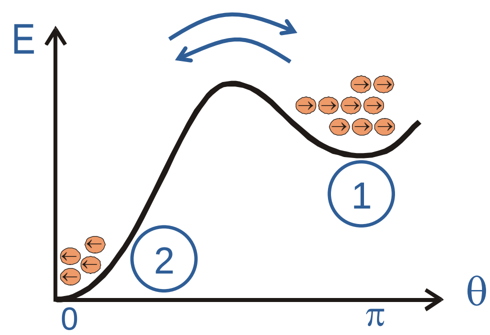
$$\tau = \tau_0 e^{-\Delta E / k_B T}$$

$\tau_0 =$ attempt period ~ 1ns

“Superparamagnetic” if $K_u V < k_B T$

Thermally Activated Reversal

What happens over time?



$$\frac{dN_{1\to 2}}{dt} \propto N_1(t)e^{-\frac{\Delta E_1}{kT}}$$

$$\frac{dN_{2\to 1}}{dt} \propto N_2(t)e^{-\frac{\Delta E_2}{kT}}$$

$$\frac{dN_1(t)}{dt} = \frac{dN_{2\to 1}}{dt} - \frac{dN_{1\to 2}}{dt}$$

$$\frac{dM(t)}{dt} = 2M_r\frac{dN_1(t)}{dt}$$

$$M(t) = 2NM_r e^{-\frac{t}{\tau}} - NM_r \quad \text{where } \tau = C^{-1}e^{\frac{\Delta E}{kT}}$$

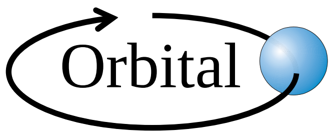
Spin Dynamics

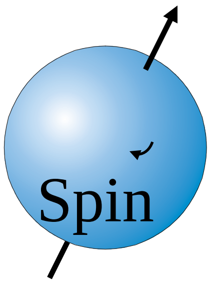
The response of magnetization to time-varying fields is affected by the angular momentum associated with spin or orbital magnetic moment.

Consider this when:

- Frequencies above ~ 1 GHz
- Pulses shorter than ~ 1 nsec

Magnetic Moment and Angular Momentum Are Linked





Angular momentum

$$L = m_e \omega r^2$$

$$L = \frac{\hbar}{2}$$

Magnetic moment

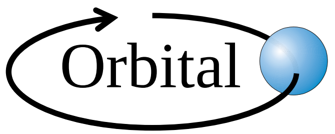
$$m = I\,A = \frac{-e\omega}{2\pi} \pi r^2$$

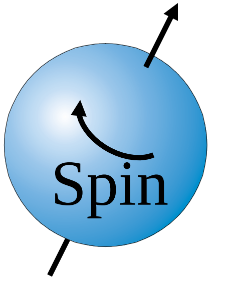
$$m_B = \frac{\hbar e}{2m_e}$$

$$\frac{m}{L} = \frac{-e}{2m_e}$$

$$\frac{m}{L} = \frac{-e}{m_e}$$

The g-factor or "gyromagnetic ratio"





$$\frac{m}{L} = \frac{-e}{2m_e}$$

$$\frac{m}{L} = \frac{-e}{m_e}$$

$$g = 1 \qquad \frac{m}{L} = g\,\frac{-e}{2m_e} \qquad g = 2$$

SI units:

$$\frac{e}{m_e} = 1.76 \cdot 10^{11}$$

$$\left[\frac{\text{coul}}{\text{kg}}\right] = \left[\frac{\text{A} \cdot \text{m}^2}{\text{kg} \cdot \text{m}^2/\text{sec}}\right] = \left[\frac{\text{rad/sec}}{\text{tesla}}\right]$$

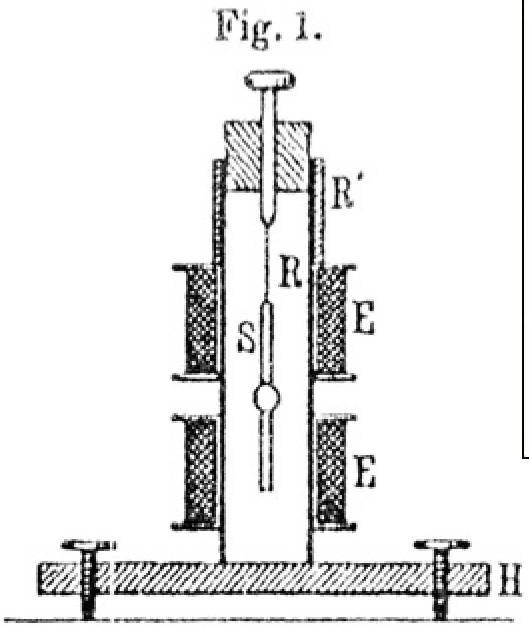
g-factors

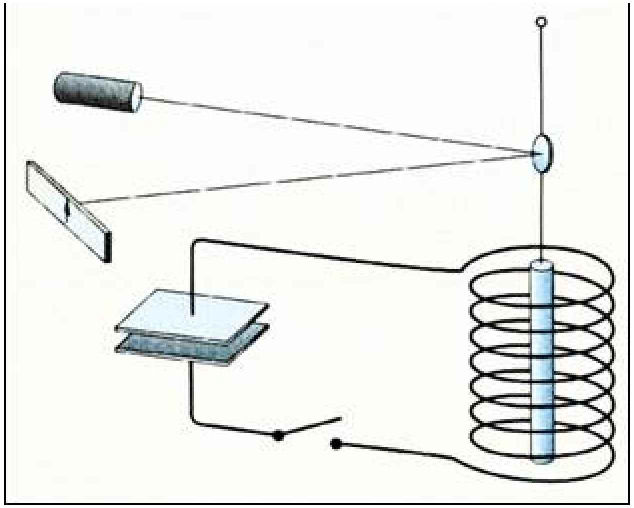
Substance	g *
Iron	1.93
Cobalt	1.85
Nickel	1.84-1.92
Magnetite	1.93
Heusler alloy	2.00
Permalloy	1.9
Supermalloy	1.91

Ferromagnetic materials have g ≈ 2
⇒ most of magnetic moment is from spin

* From Kittel via Chikazumi's Physics of Magnetism

Einstein-DeHaas Experiment (1915)





$$\frac{m}{L} = g\,\frac{-e}{2m_e}$$

$$\Delta m = g\,\frac{-e}{2m_e}\,\Delta L$$

$$\Delta m = 2M_S\,Vol$$

The gyromagnetic “constant”, $\boldsymbol{\gamma}$

$$\gamma \equiv \frac{m}{L} = g\,\frac{-e}{2m_e}$$

For most materials:

$$\gamma = 1.76 \times 10^{11}\,\frac{rad/s}{tesla} = 28\,\frac{GHz}{tesla}$$

Spin Dynamics: The Landau-Lifschitz Equation

Torque on a magnetic moment:

$$\vec{T} = \vec{m} \times \vec{B} = \vec{m} \times \mu_0 \vec{H}$$

The gyroscope equation:

$$\frac{d\vec{L}}{dt} = \vec{T}$$

The gyromagnetic relation:

$$\vec{m} = -\gamma \vec{L}$$

and

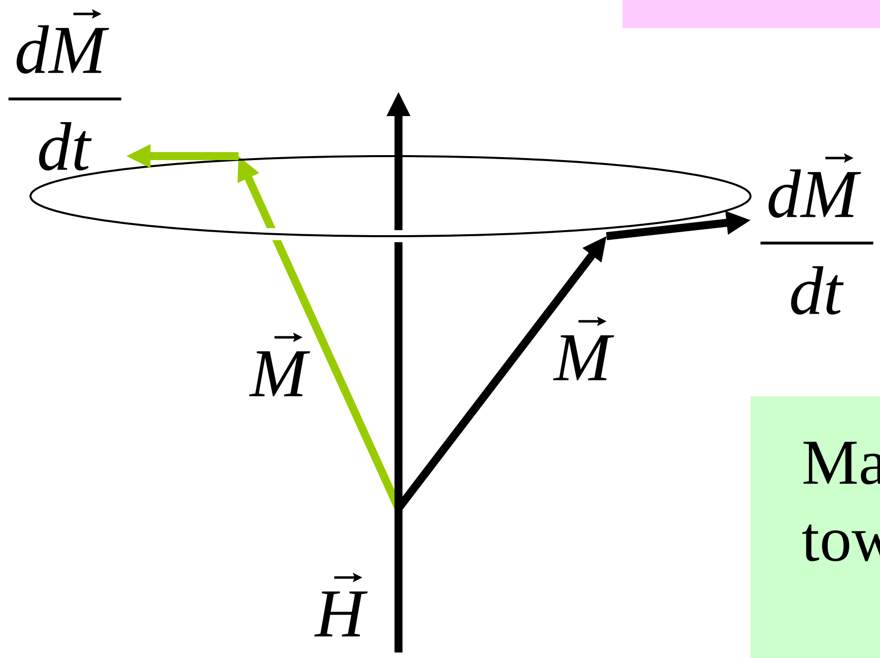
$$\vec{M} = \vec{m} / volume$$

Gives:

$$\frac{d\vec{M}}{dt} = -\mu_0 \gamma \vec{M} \times \vec{H}$$

Spin Precession

$$\frac{d\vec{M}}{dt} = -\mu_0 \gamma \vec{M} \times \vec{H}$$



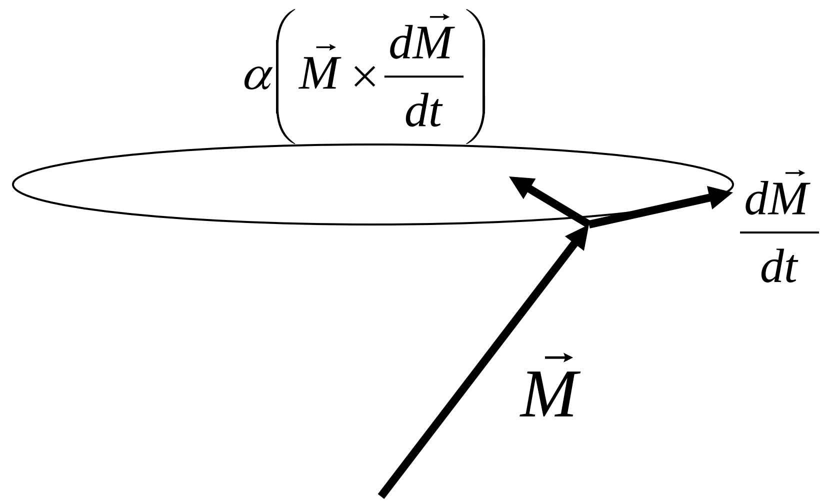
Magnetization never turns towards *H*, just precesses at frequency $28\frac{GHz}{tesla}$

$\omega = \gamma \mu_0 H$ $\qquad f = \frac{1}{2\pi} \gamma \mu_0 H$

Landau-Lifshitz Equation with Gilbert Damping

$$\frac{d\vec{M}}{dt} = -\mu_0\gamma\left(\vec{M}\times\vec{H}\right) + \frac{\alpha}{|M|}\left(\vec{M}\times\frac{d\vec{M}}{dt}\right)$$

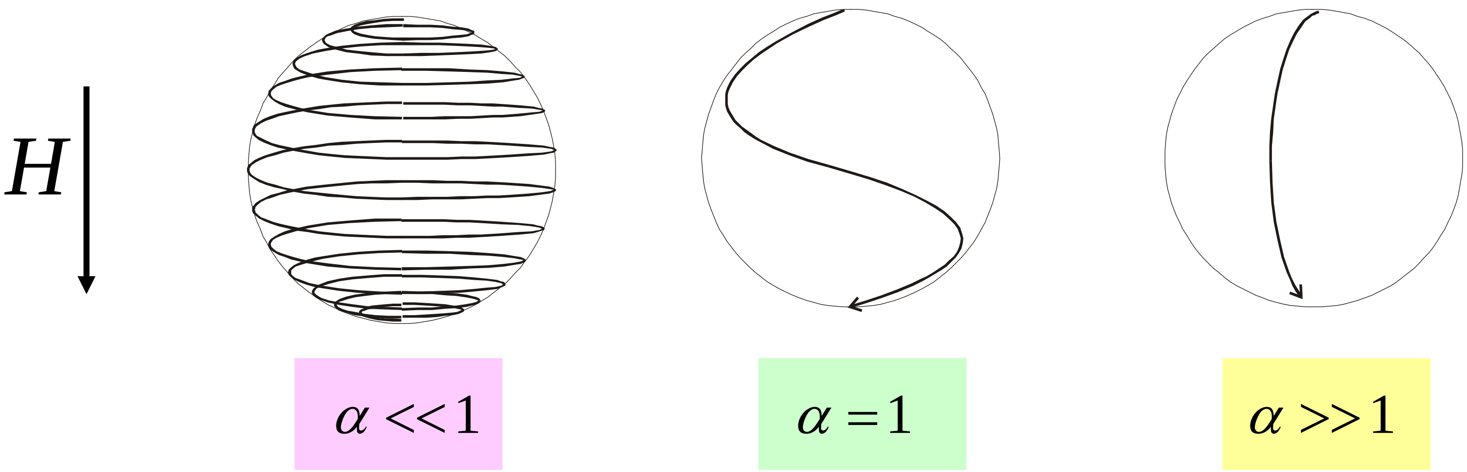
$\boldsymbol{\alpha}$ is the Gilbert damping parameter



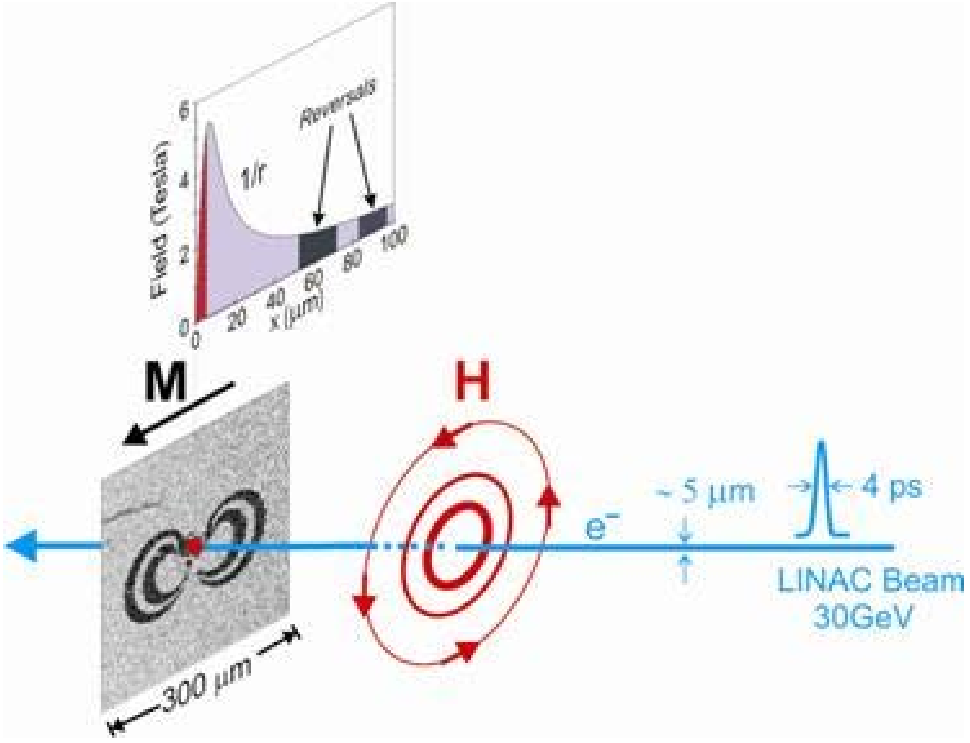
Equivalent: $$\frac{d\vec{M}}{dt} = -\mu_0\gamma'\left(\vec{M}\times\vec{H}\right) + \frac{\lambda}{|M|}\left[\vec{M}\times\left(\vec{M}\times\vec{H}\right)\right]$$

Reversal with Damped Precession

Upon reversal of field, magnetization vector follows a complex path.

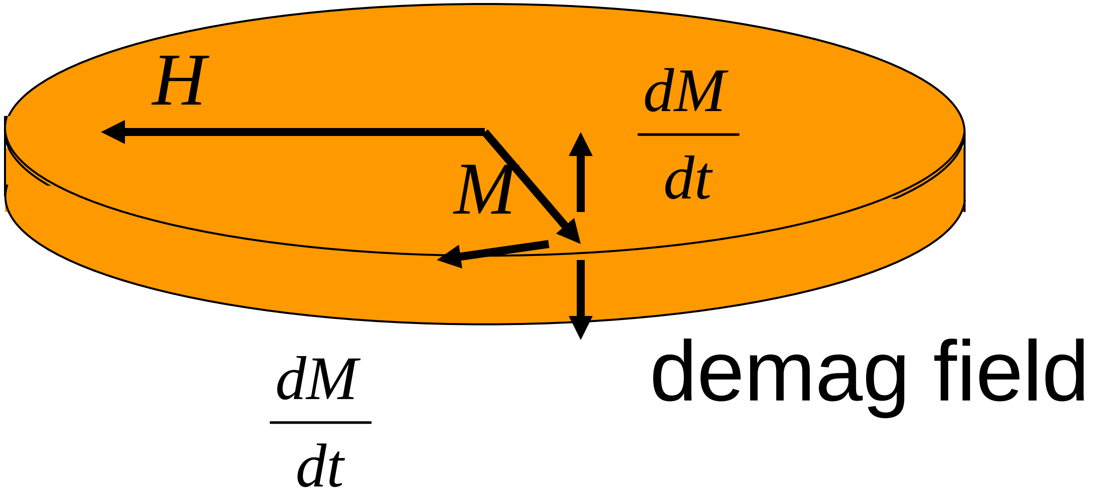


Precessional Switching



Reversal in a Thin Film

- M tries to precess out of film plane
- Demagnetizing field pushes back
- Demag. field increases effective field



Kittel relation: $H_{eff} = \sqrt{H(H+M)}$

Ferromagnetic Resonance (FMR)