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2014 IEEE Magnetics Society Distinguished Lecture Series

Opportunities and Challenges in Two-Dimensional Magnetic Recording

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TDMR Talk Overview

- Introduction
- Magnetic Recording Models in Two Dimensions
- Testing in Two Dimensions
- Magnetic Signal Processing in Two Dimensions
- Questions

A Hard Disk Drive

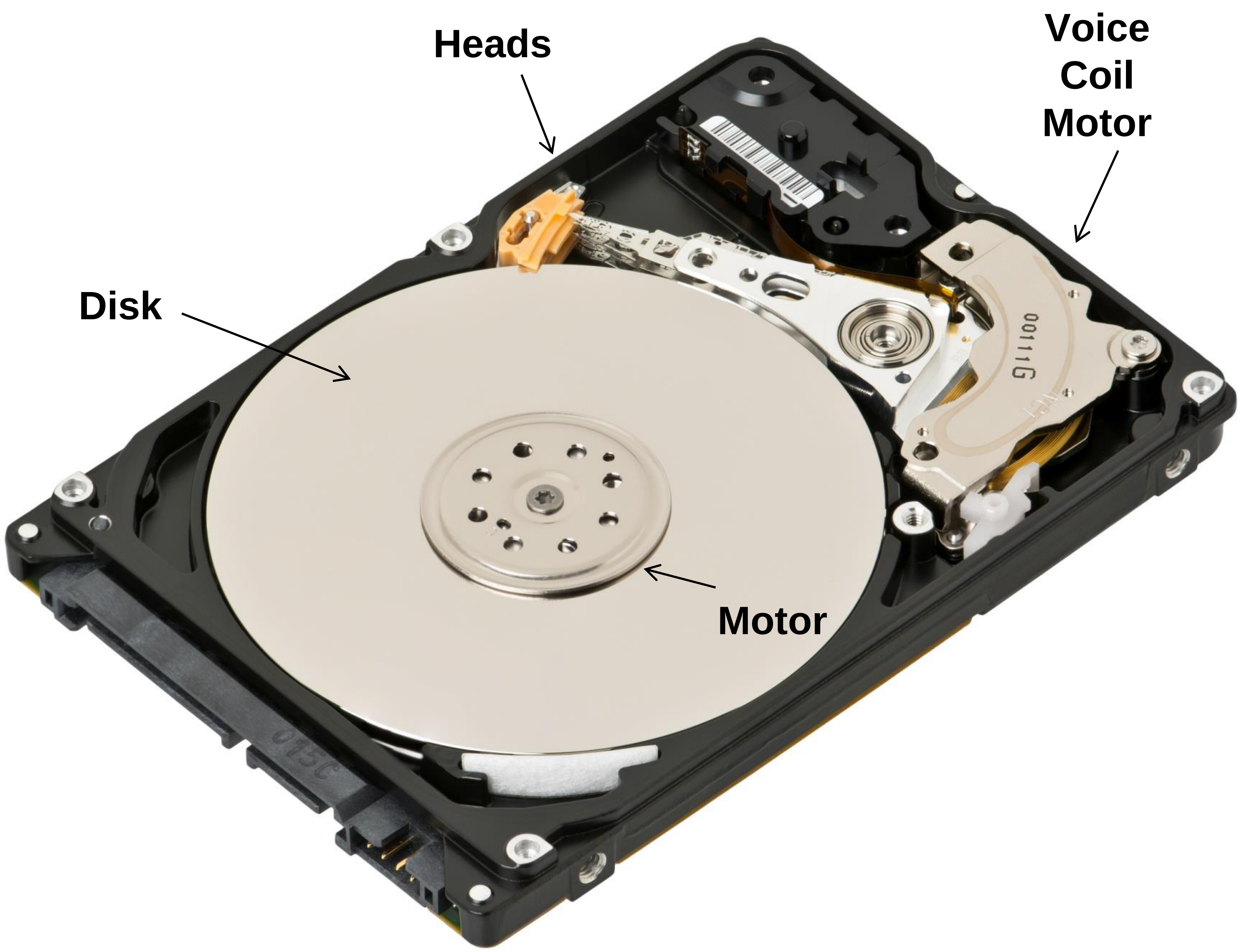
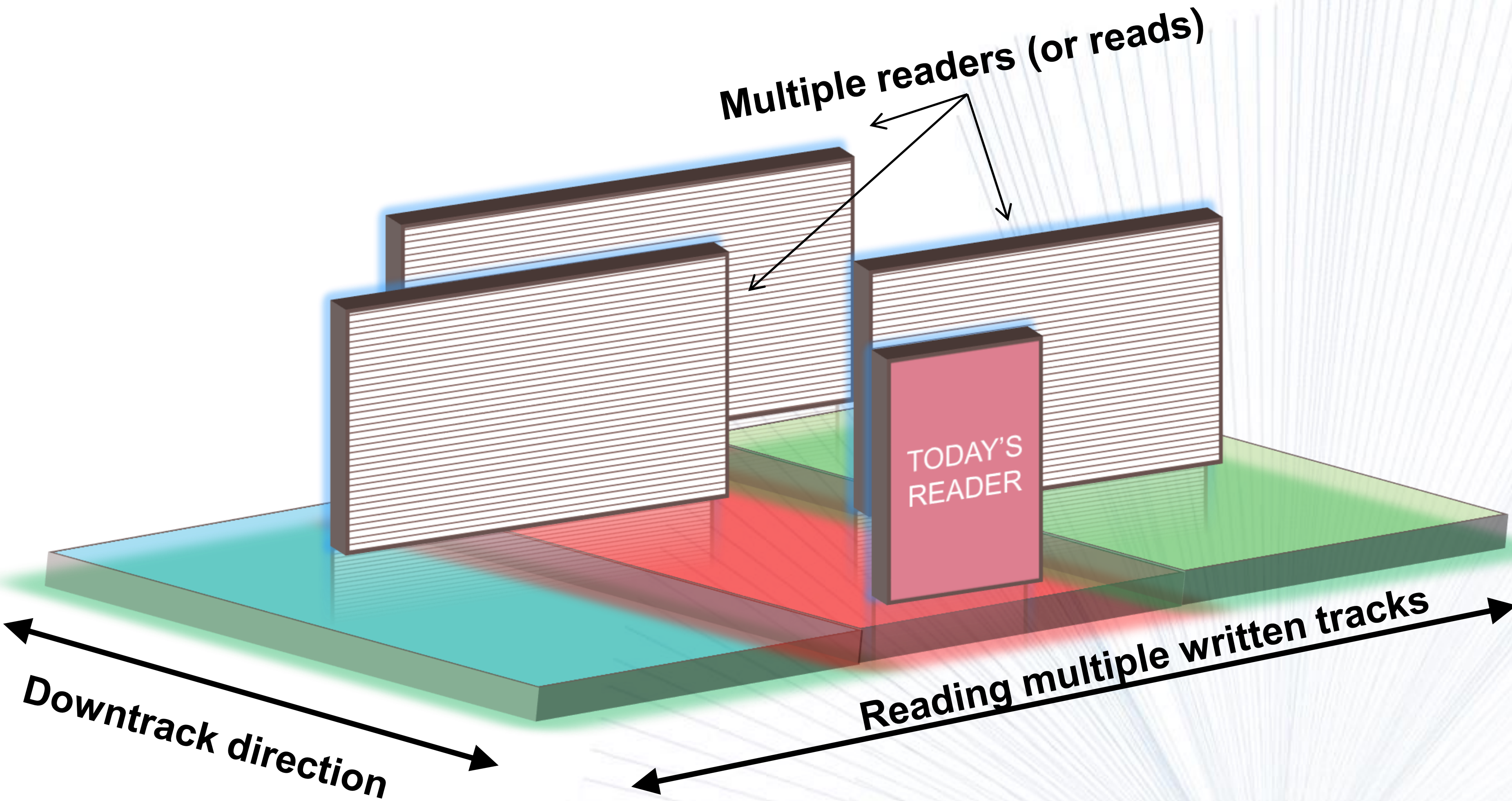


Image from wikipedia.com

An (old) Recording Head on a Disk

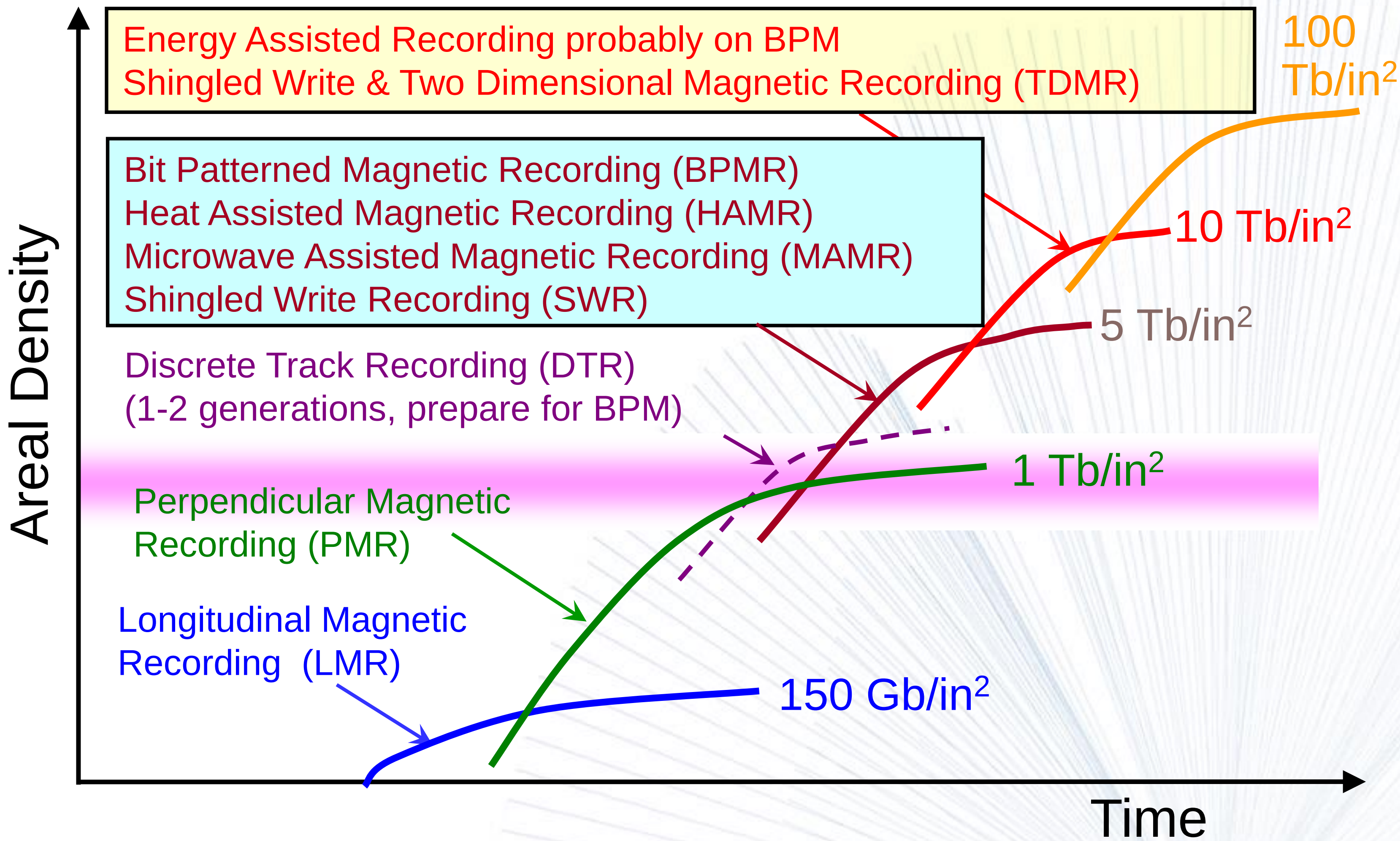
Image from wikipedia.com

A TDMR Perspective



Detecting data in this configuration is a multidisciplinary 2D problem.

A 2009 Industry Technology Roadmap

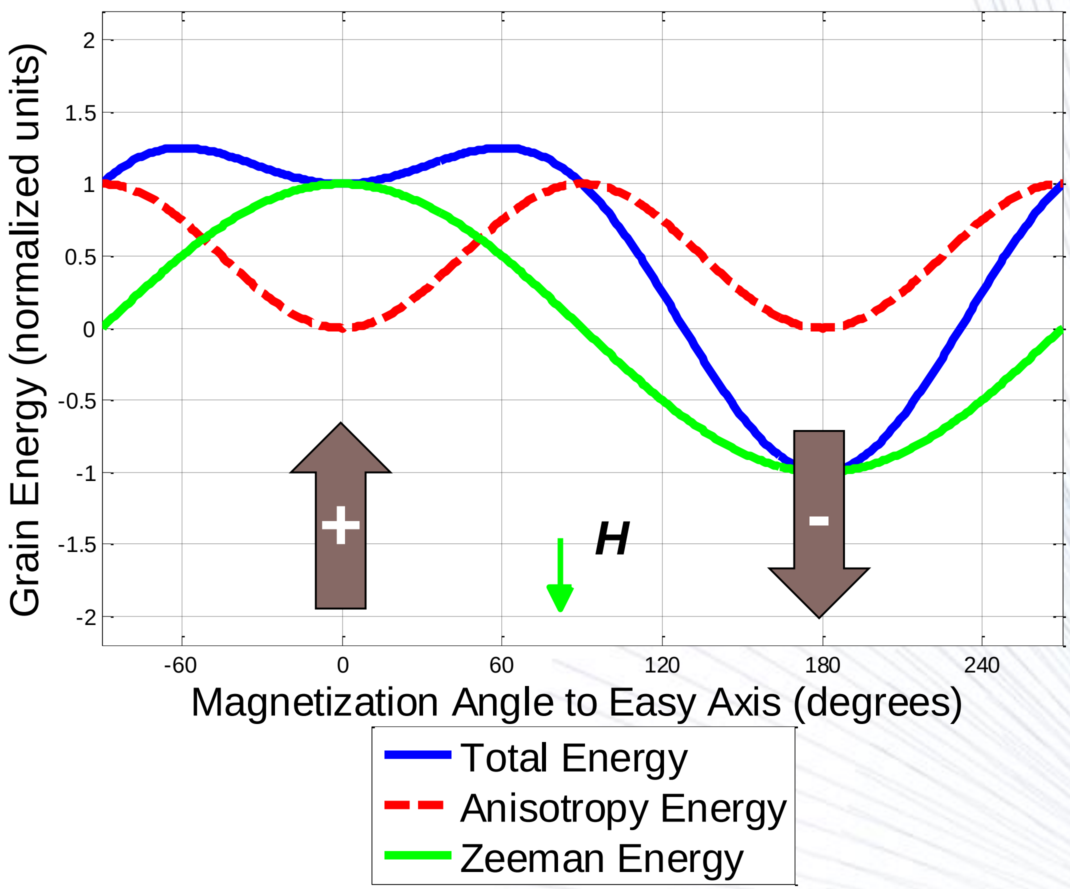


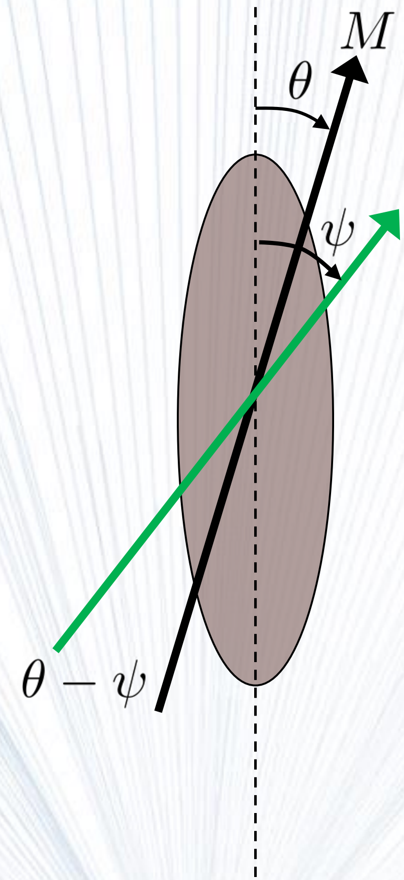
Further reading: Y. Shiroishi, Intermag 2009, FA-01

Magnetic Energy Relations in Small Symmetric Particles

ANISOTROPY ZEEMAN THERMAL

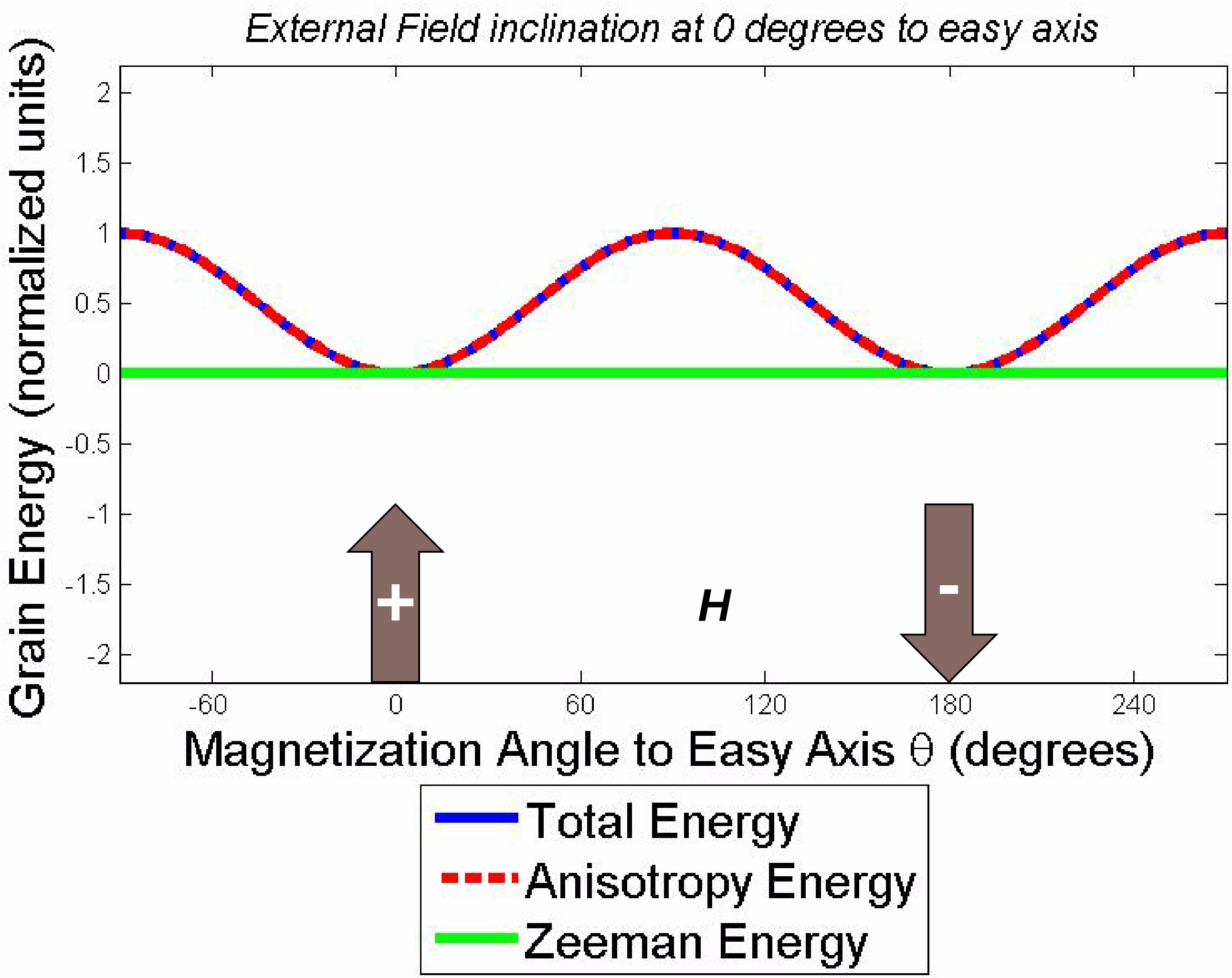
$$E = \overbrace{K_u V \sin^2(\theta) + \ldots}^{\text{ANISOTROPY}} - HMV\cos(\theta - \psi) + E_T$$





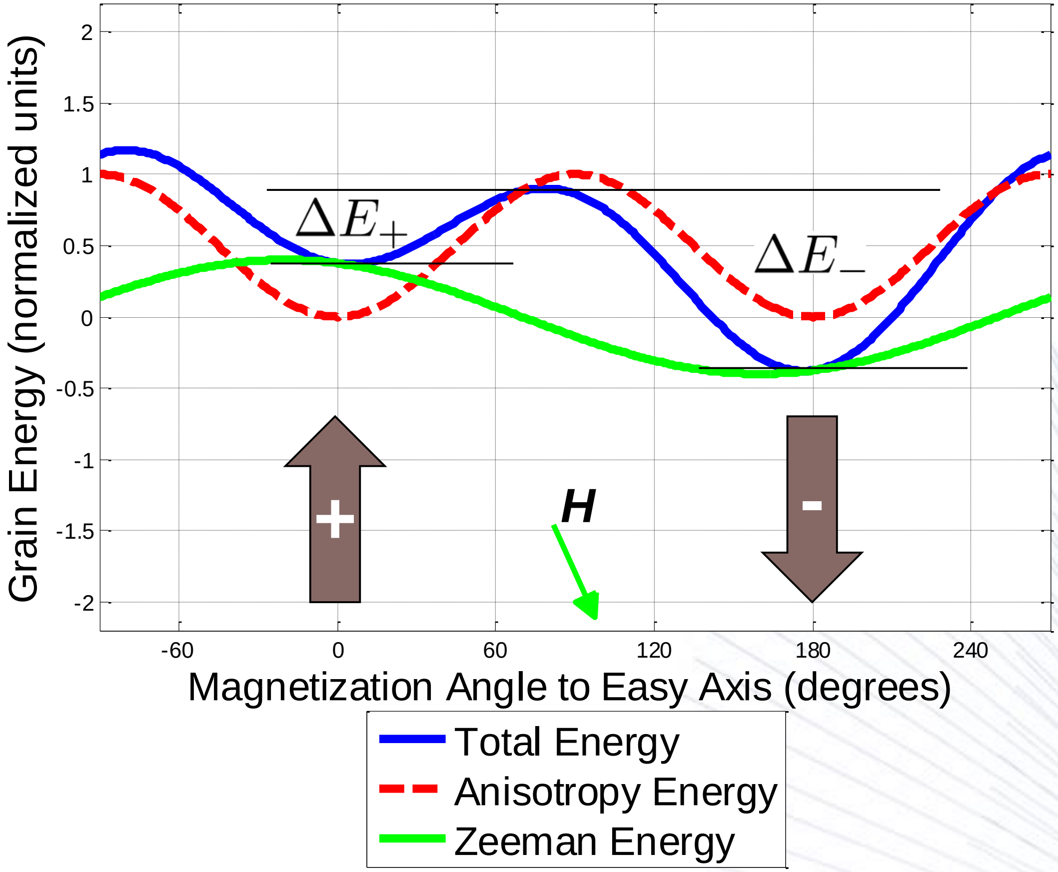
Suggested reading: Zhang and Bertram, *IEEE Trans. on Magnetics,* Vol 34, No. 5 1998
R. Wood, *IEEE Trans. on Magnetics,* Vol. 41, No. 1, 2009

A View with Dynamic External Field *H*



Energy Barriers and the Probability of Switching

$$\Delta E_{\pm} \approx K_u V (1 \mp H/H_n)^{\alpha}$$



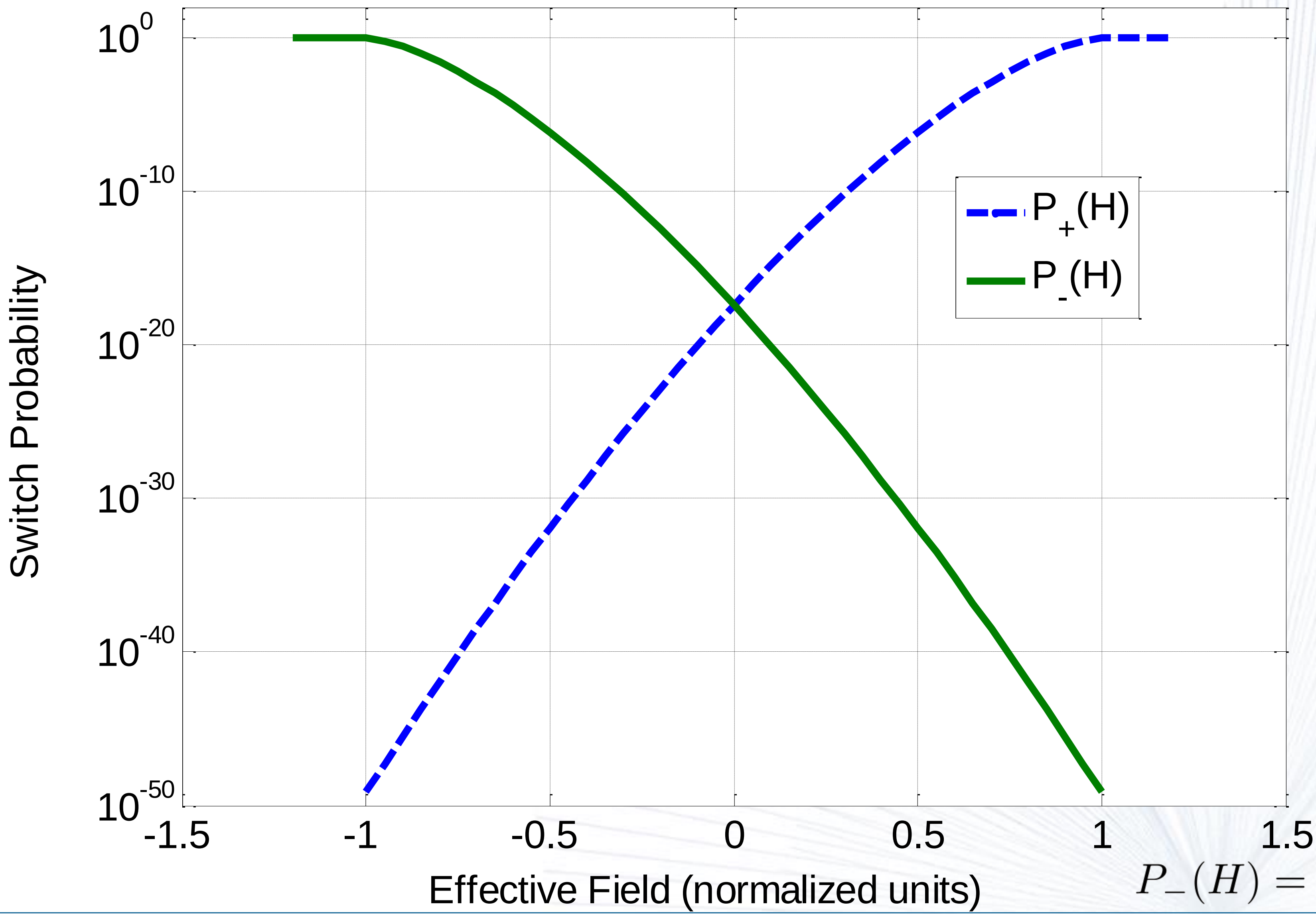
$$P_{\pm}(H) \approx e^{-\Delta E_{\pm}/kT}$$

Probability that a grain switches in a given moment $\tau_0 = 1/f_0$

Suggested reading: Zhang and Bertram, *IEEE Trans. on Magnetics,* Vol 34, No. 5 1998
Victora and Chen,, *Proc. of the IEEE,* Vol. 96, No. 11, Nov 2008
Victora, *Physical Review Letters,* Vol. 63, No. 4, July 1989

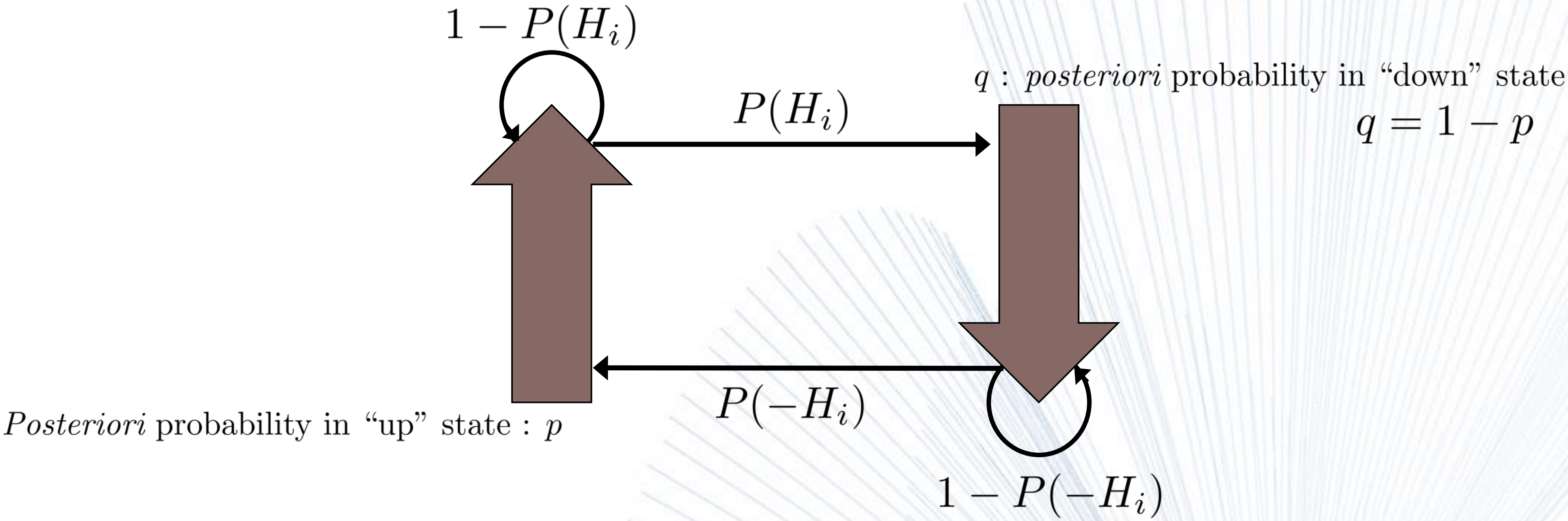
Exemplary Switch Probabilities vs. Field Strength

Plotted for $K_u V / kT = 40$



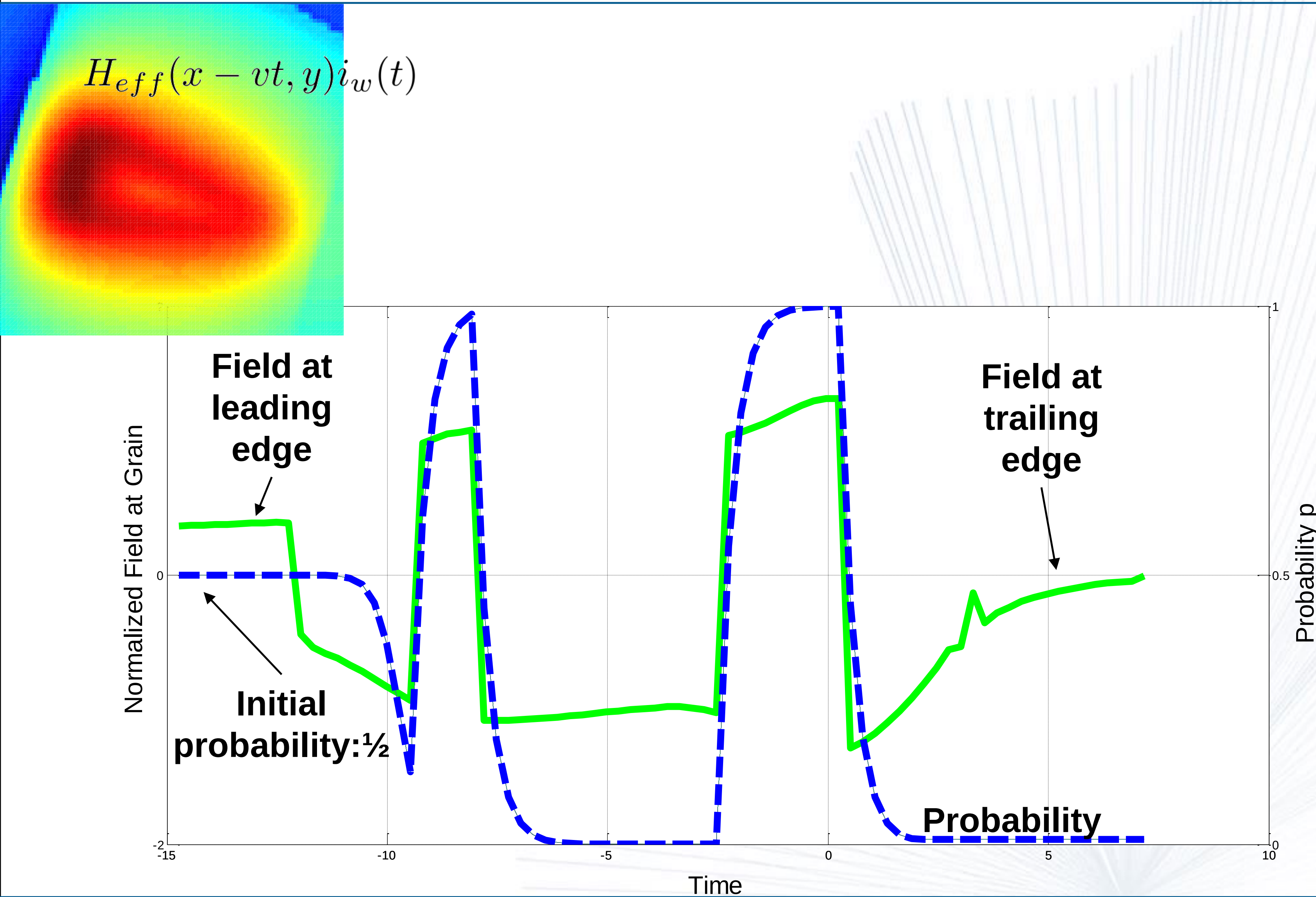
$$P_-(H) = P_+(-H)$$

A Discrete Time Markov Model for Two-State Magnetization

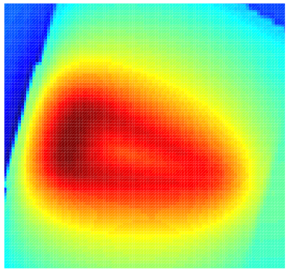


$$\begin{pmatrix} p_{i+1} \\ q_{i+1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 - P(H_i) & P(-H_i) \\ P(H_i) & 1 - P(-H_i) \end{pmatrix}}_{\mathbf{T}(H_i)} \begin{pmatrix} p_i \\ q_i \end{pmatrix}$$

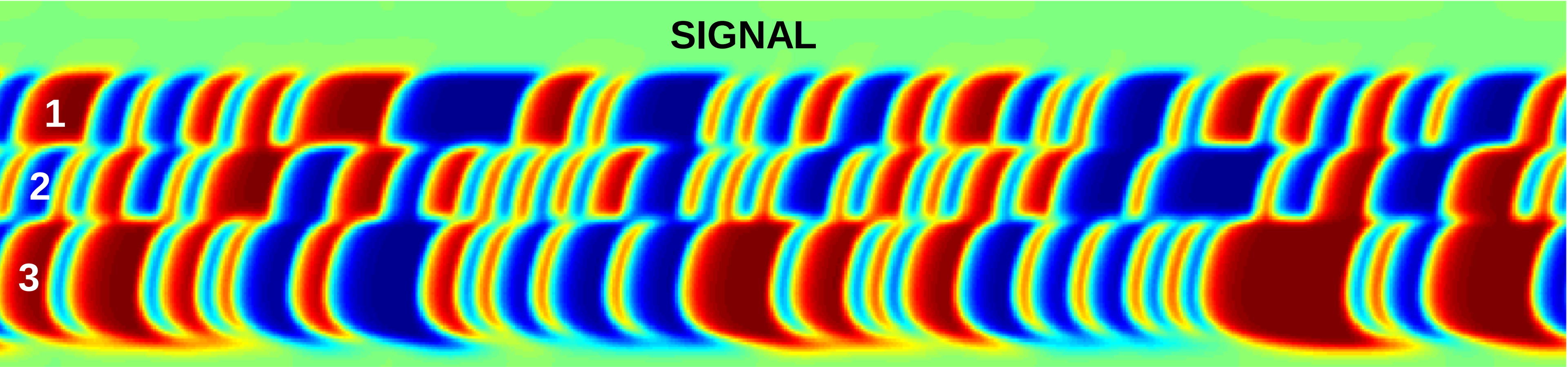
An example of probability changes at a grain over time



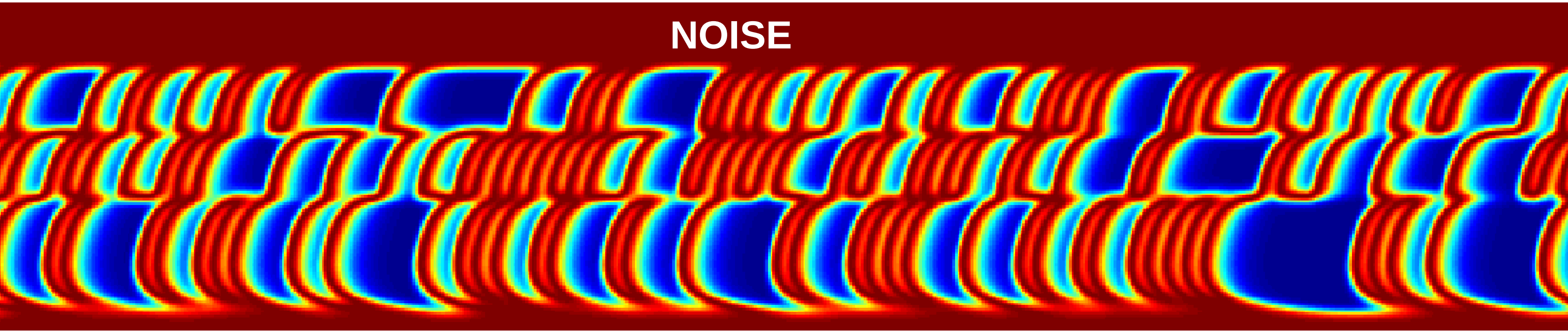
Probability and Noise Maps in Two Dimensions



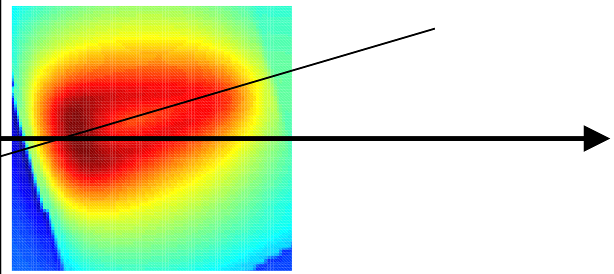
$$H_{eff}(x - vt, y)i_w(t)$$

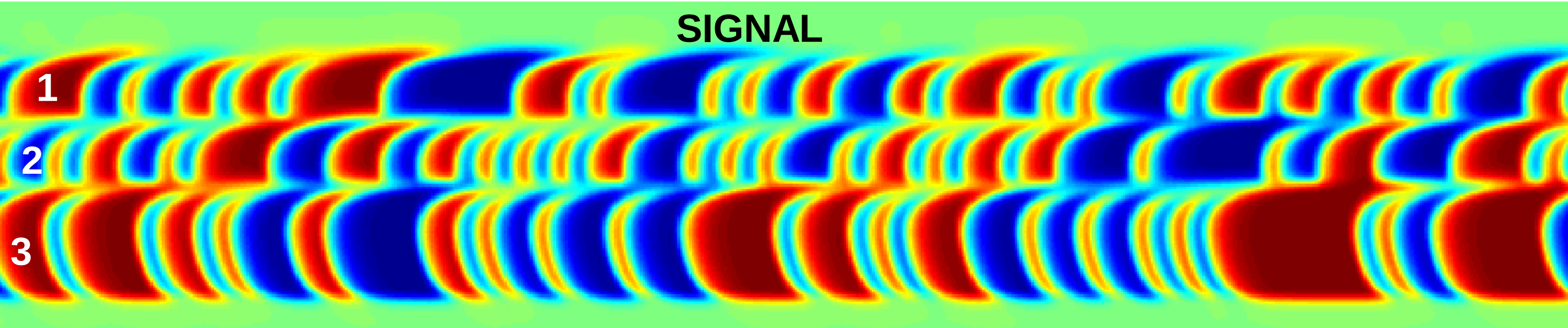


$$\sigma_M^2(x,y) = 4(1 - p(x,y))p(x,y) \quad \bar{M}(x,y) = 2p(x,y) - 1$$



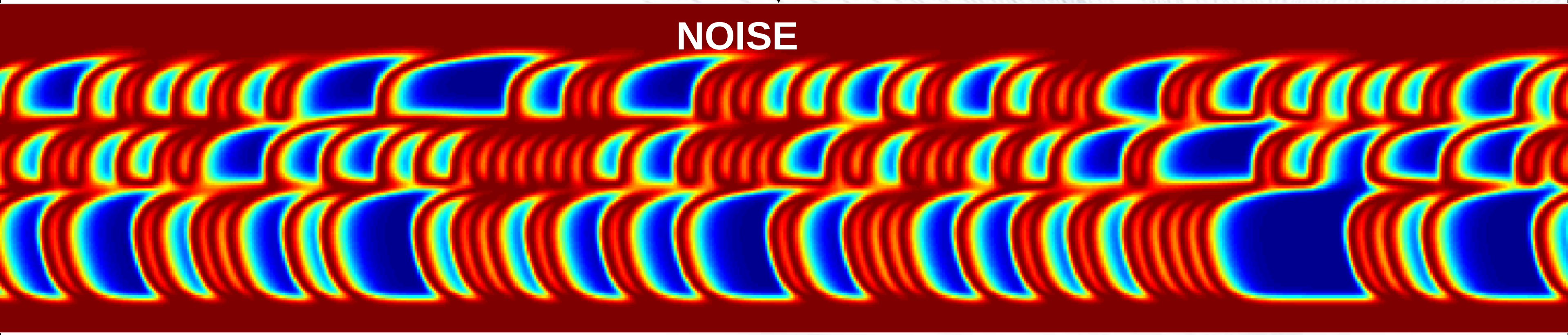
Wrong-Way Shingling with Different Head Skew



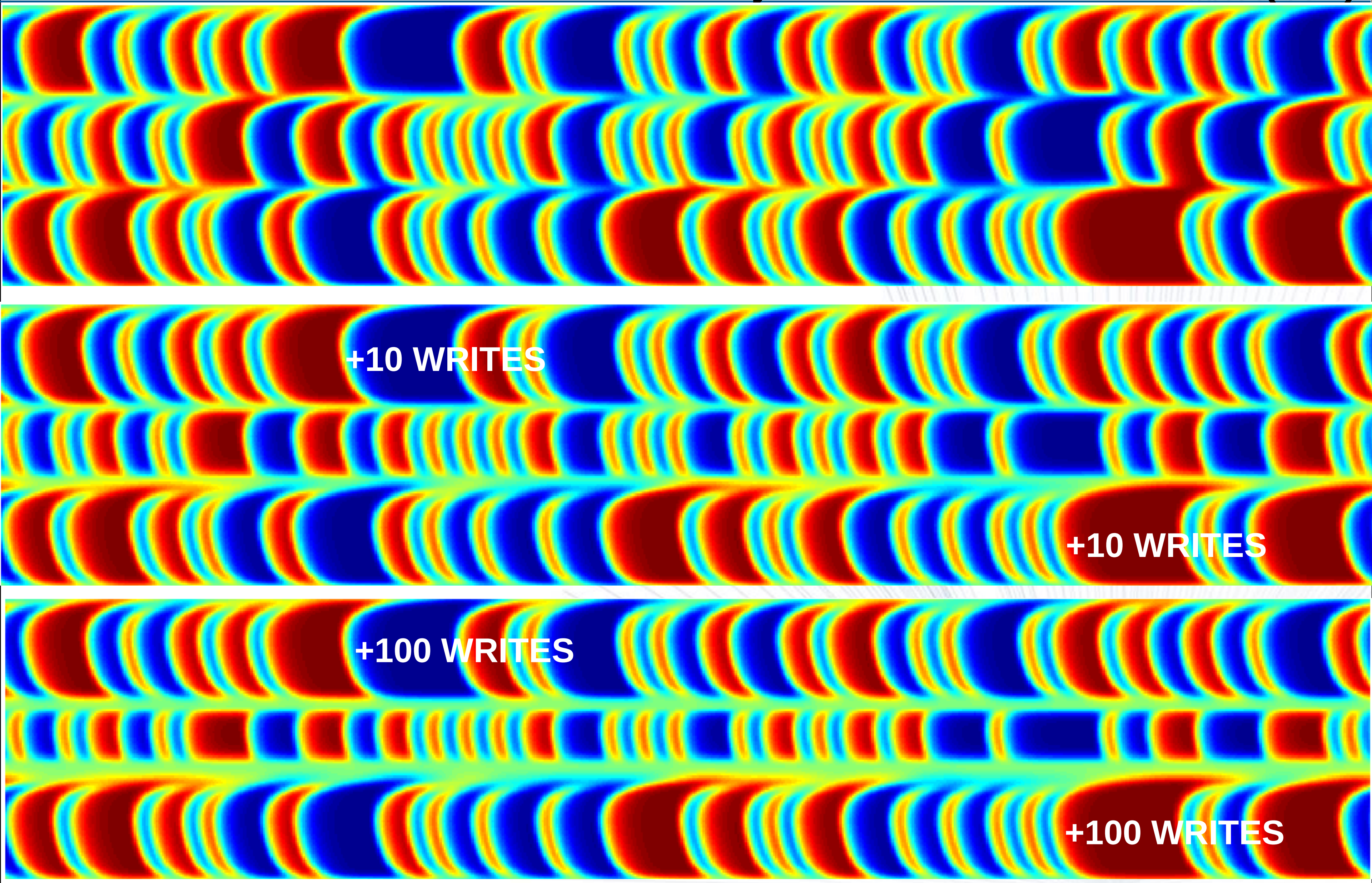


$$\sigma_M^2(x,y) = 4(1-p(x,y))p(x,y)$$

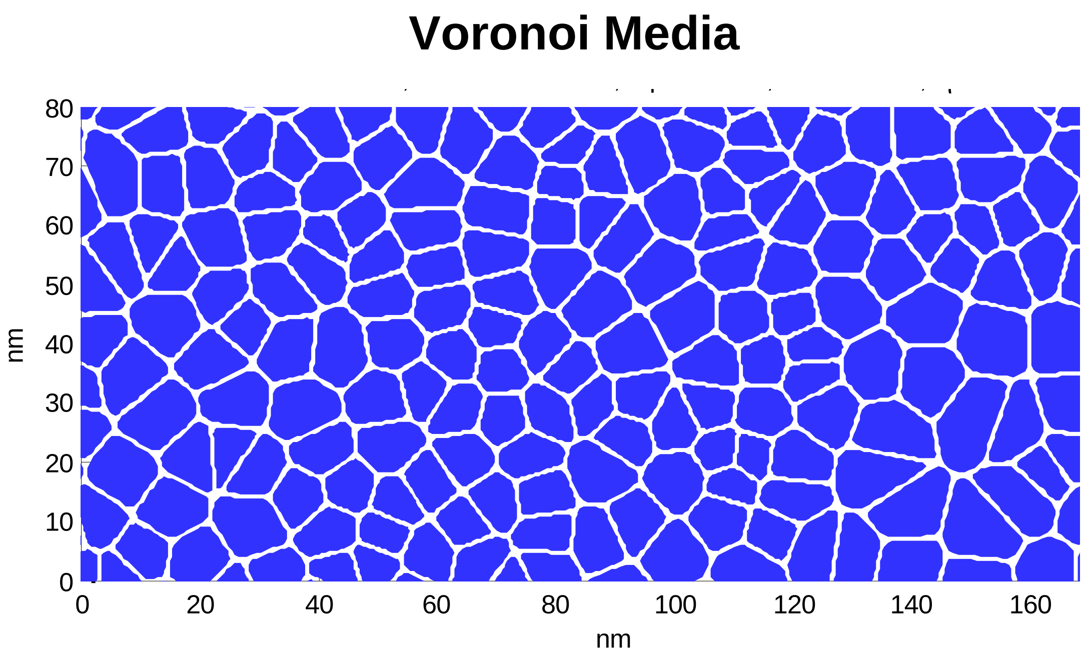
$$\bar{M}(x,y) = 2p(x,y) - 1$$

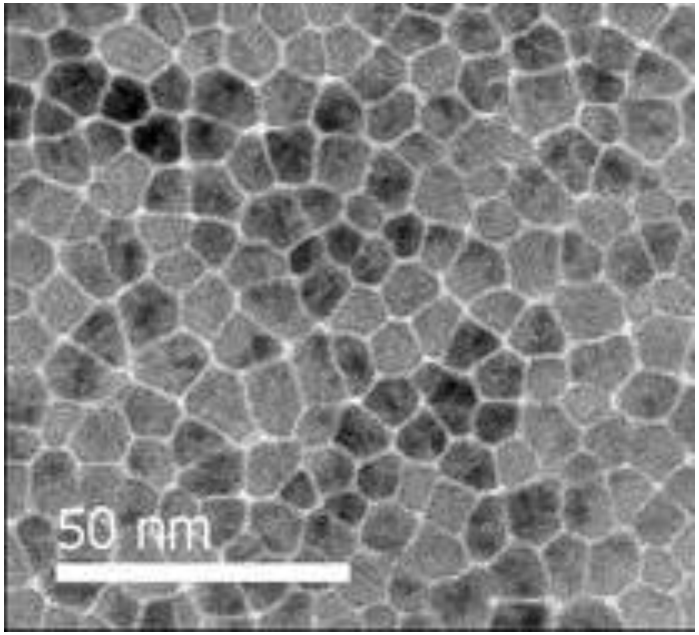


Standard PMR probability maps with Adjacent Track Interference (ATI)



(but: grains have finite and nonconstant volume)





Suggested reading: Chan and Elidrissi, *IEEE Trans. on Magnetics,* Vol. 49, Issue 6, 2013

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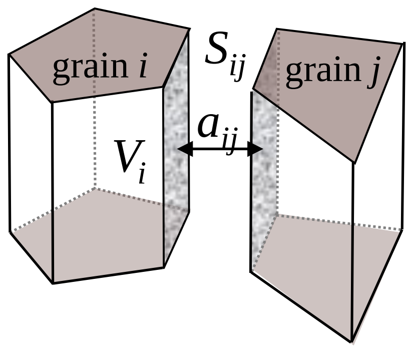
(some further complications: grains *do* interact)

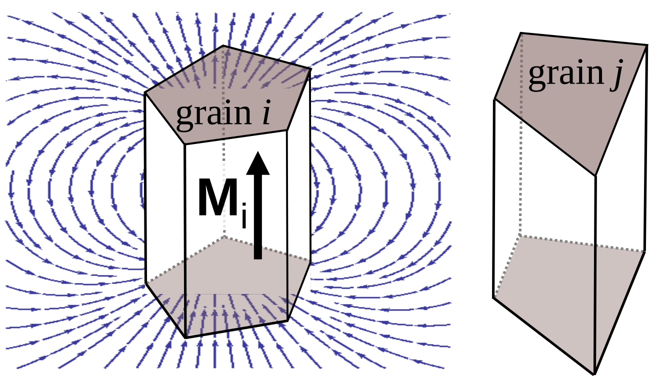
Intergranular Exchange

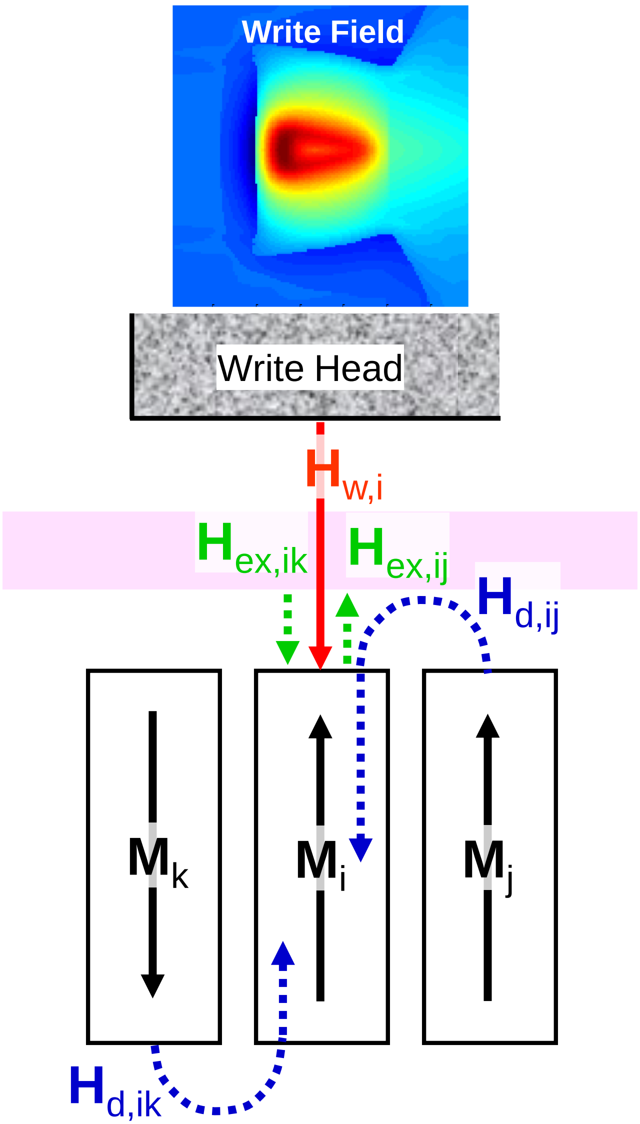
Neighbors tend to magnetize in *same* direction



Demag Fields

Neighbors tend to magnetize in *opposite* direction

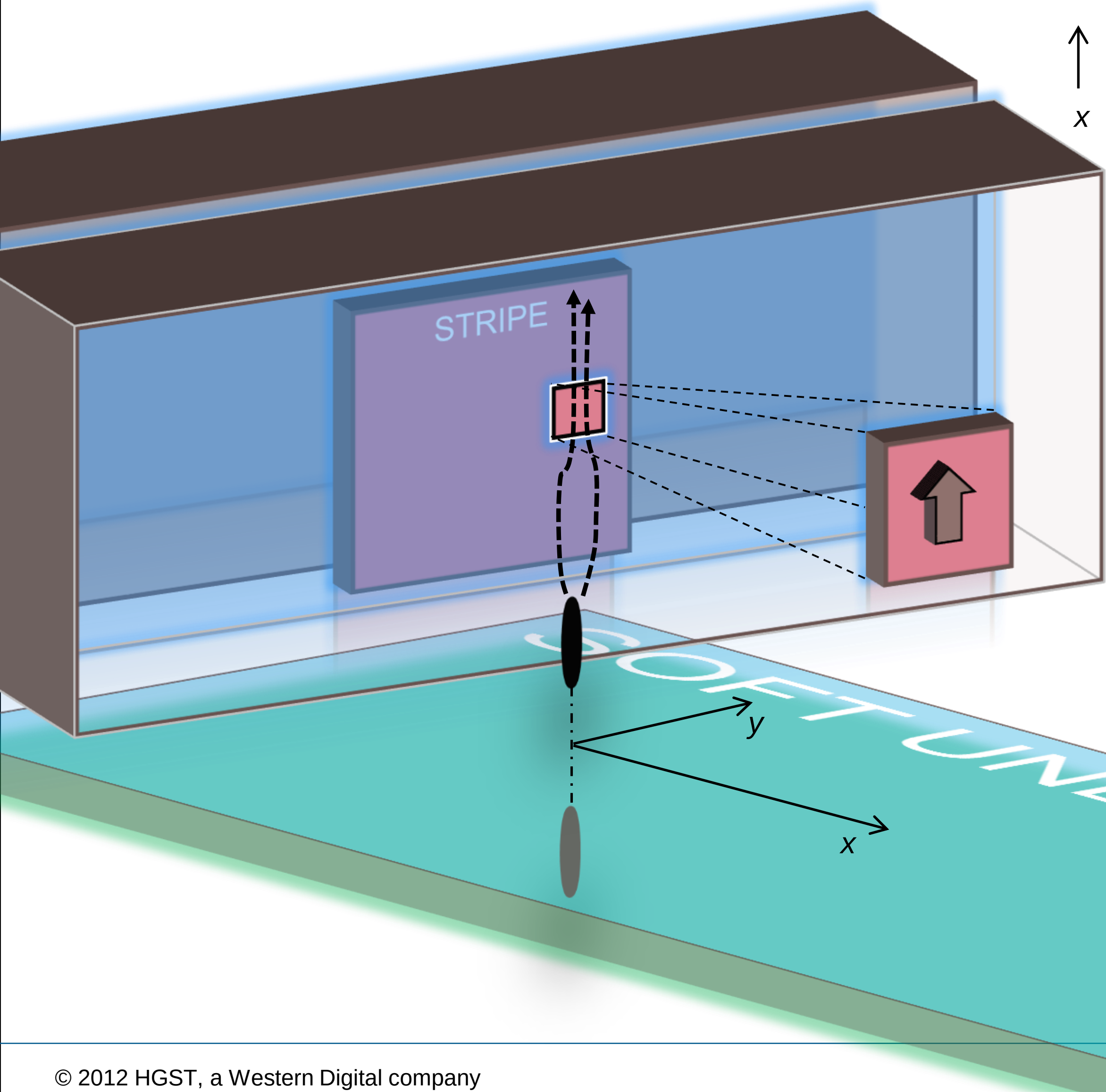


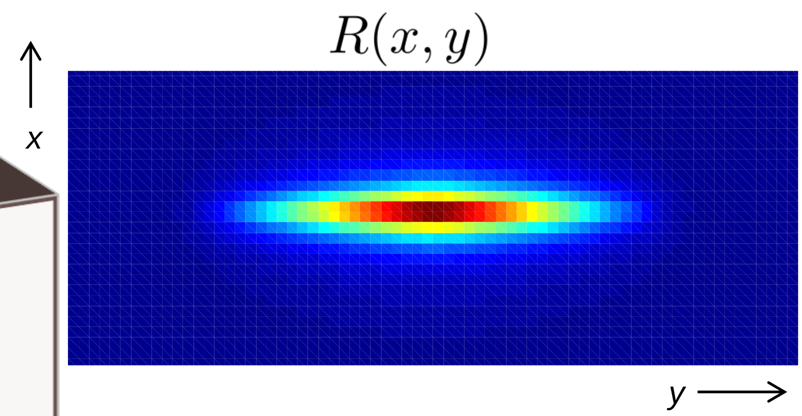


Suggested reading: S. J. Greaves et al., *Journal of Magnetism and Magnetic Materials* 287, 2005.

The Read Sensitivity or “Impulse Response” Function

Suggested reading: Yuan and Bertram, *IEEE Trans. on Magnetics,* Vol. 30, Issue 3, 1994
Wood and Wilton, *IEEE Trans. on Magnetics,* Vol. 44, Issue 7, 2008



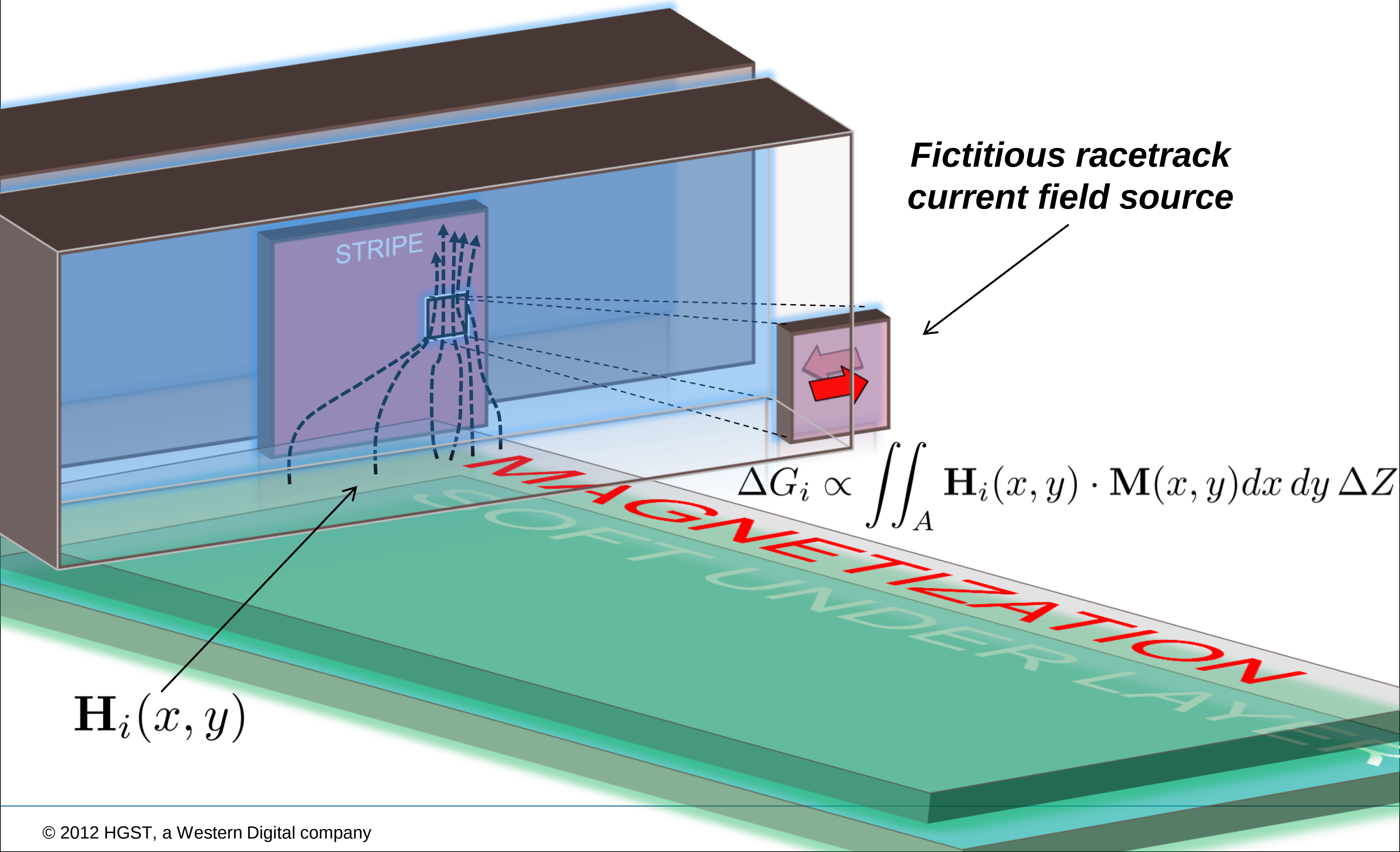


$$\Delta G_i \approx \alpha B_{Zi} + \ldots$$

$$\Delta G_{TOT} \approx \alpha \sum_i B_{Zi} + \ldots$$

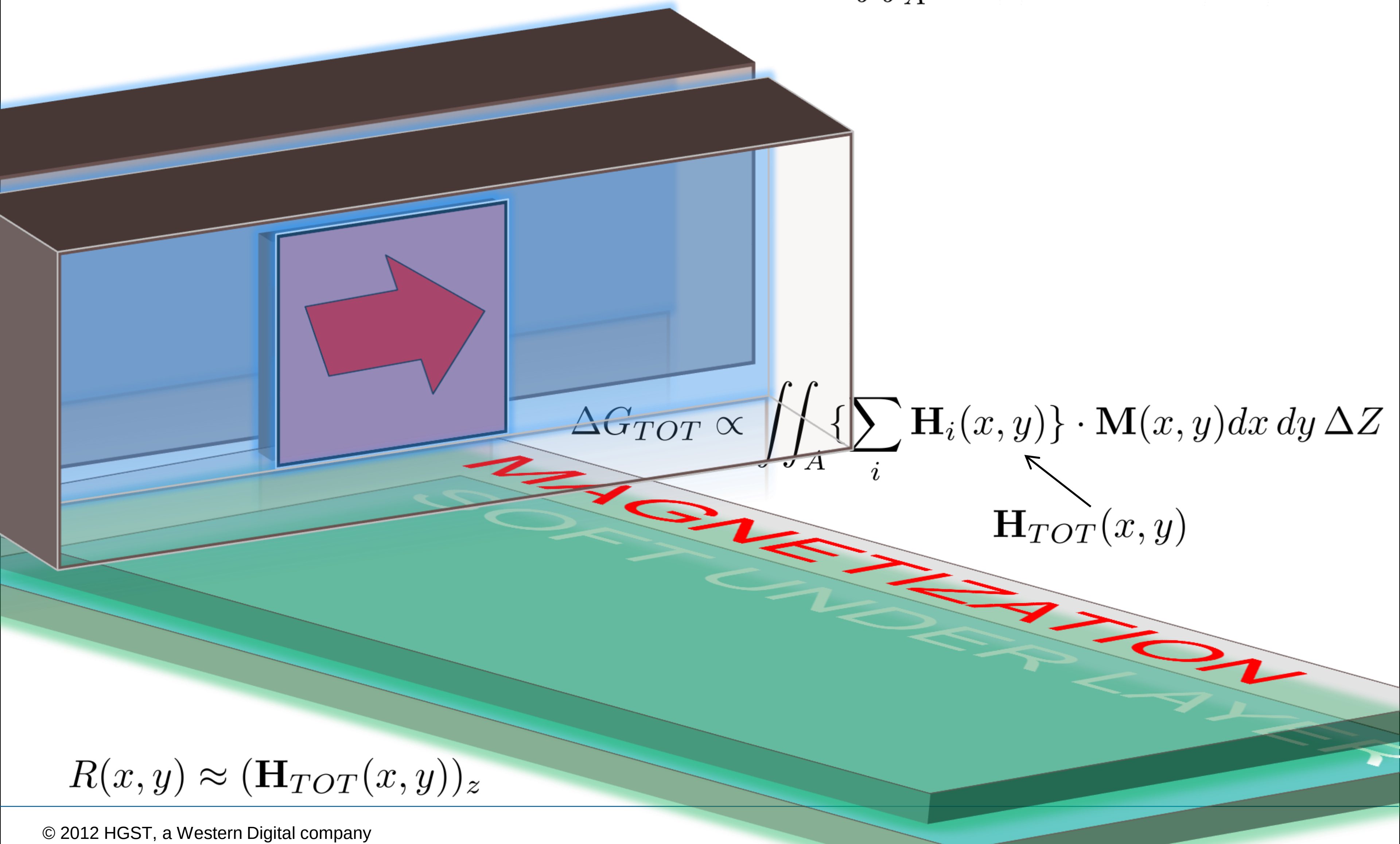
A Reciprocity Principle

Suggested reading: N. Smith, *IEEE Trans. on Magnetics,* Vol. 29, No. 5, 1993
Litvinov and Khizroev, *Journal of Applied Physics,* Vol. 97 , 2005



A Further Simplification

$$\Delta G_i \propto \iint_A \mathbf{H}(x,y) \cdot \mathbf{M}(x,y) \mathrm{d}x\, \mathrm{d}y\, \Delta Z$$

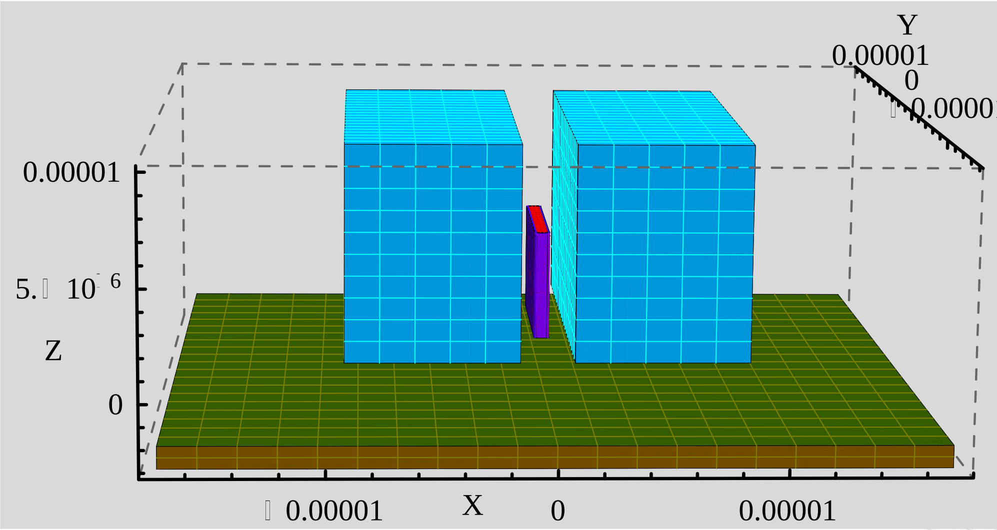


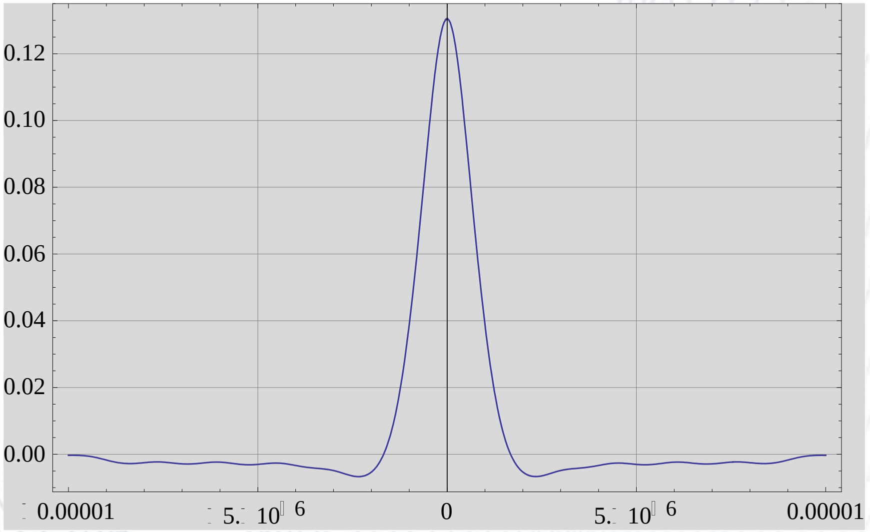
$$\Delta G_{TOT} \propto \iint_A \{\sum_i \mathbf{H}_i(x,y)\} \cdot \mathbf{M}(x,y) dx\, dy\, \Delta Z$$

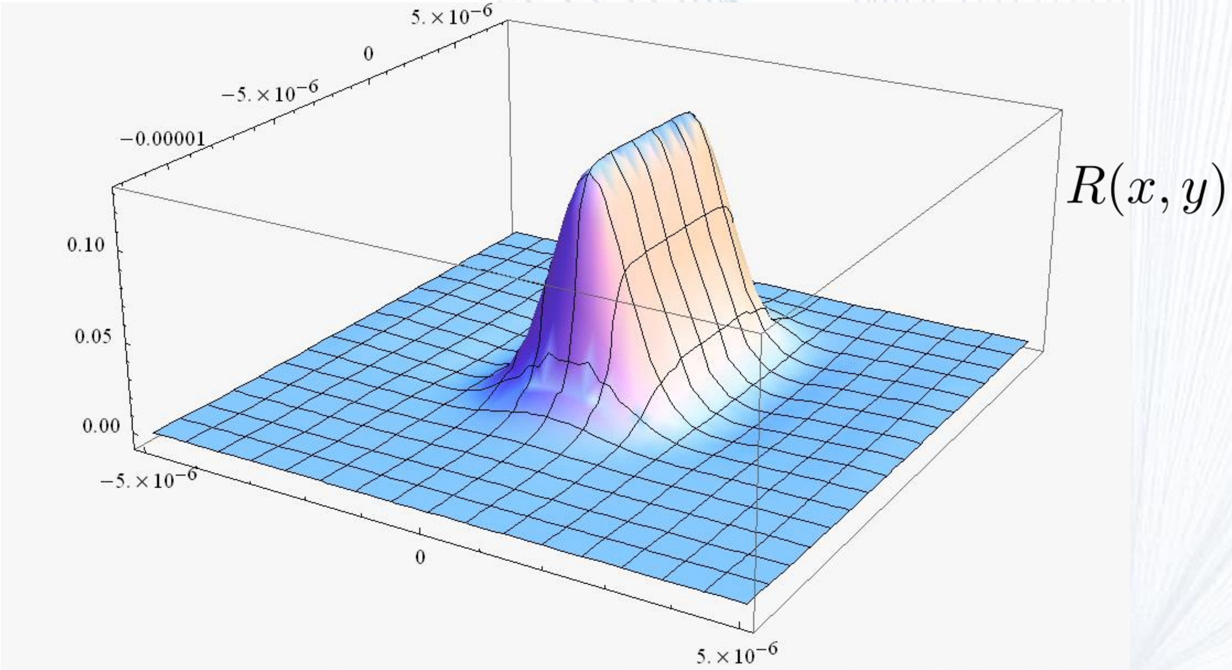
$\mathbf{H}_{TOT}(x,y)$

$$R(x,y) \approx (\mathbf{H}_{TOT}(x,y))_z$$

Numerical Results for the Head Sensitivity Function

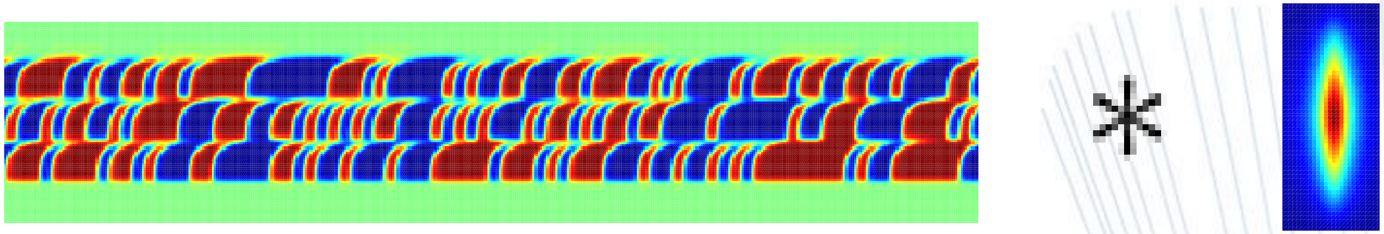




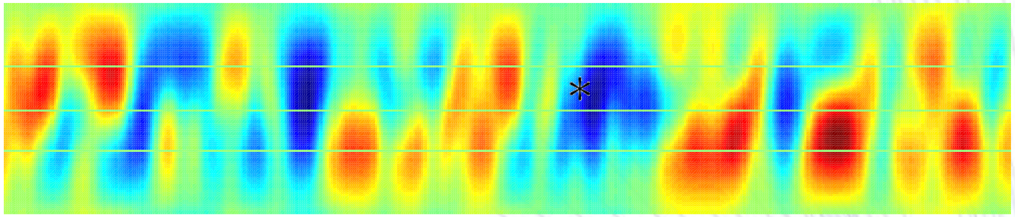


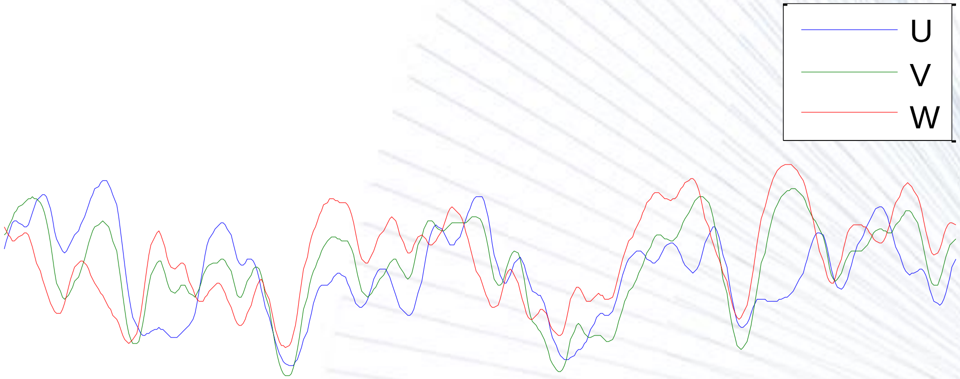
Putting it all together for linear magnetic systems: generate readback signals by 2D convolution

$\bar{M}(x,y)$ $\quad * \quad$ $R(x,y)$



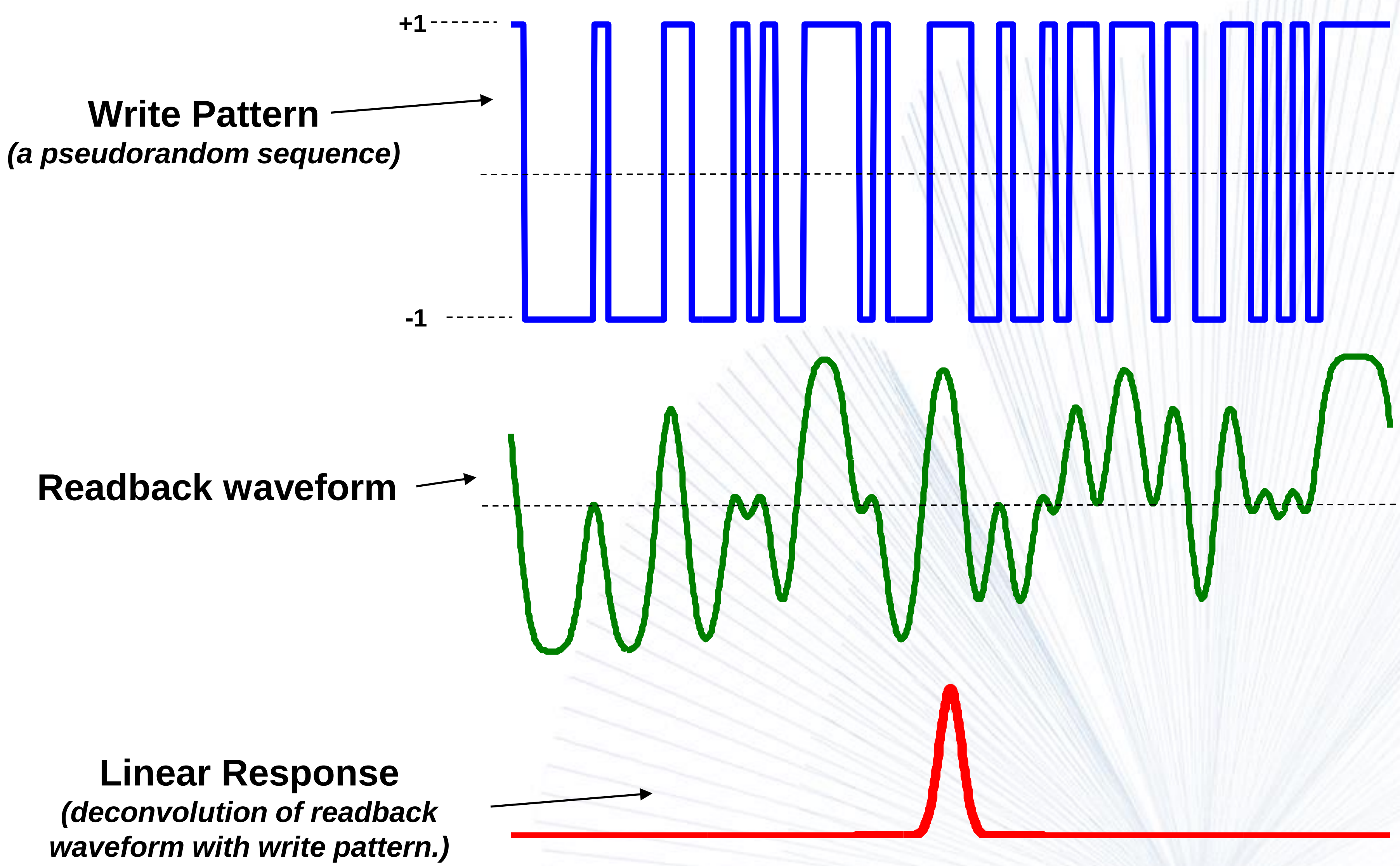
$= X(x,y)$



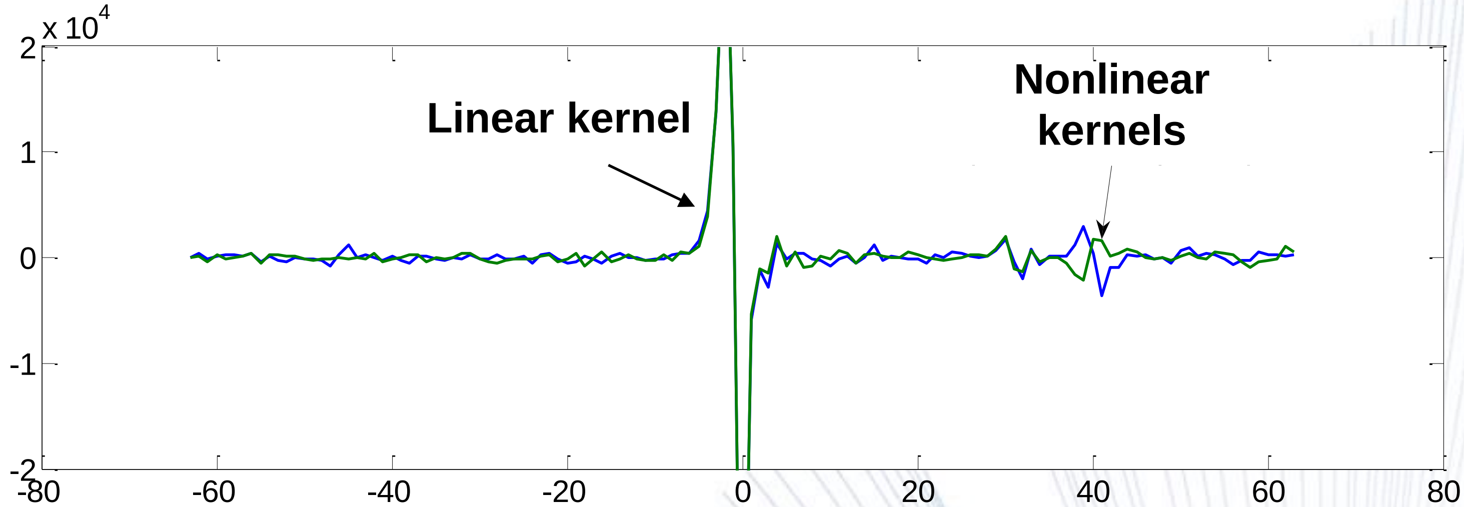


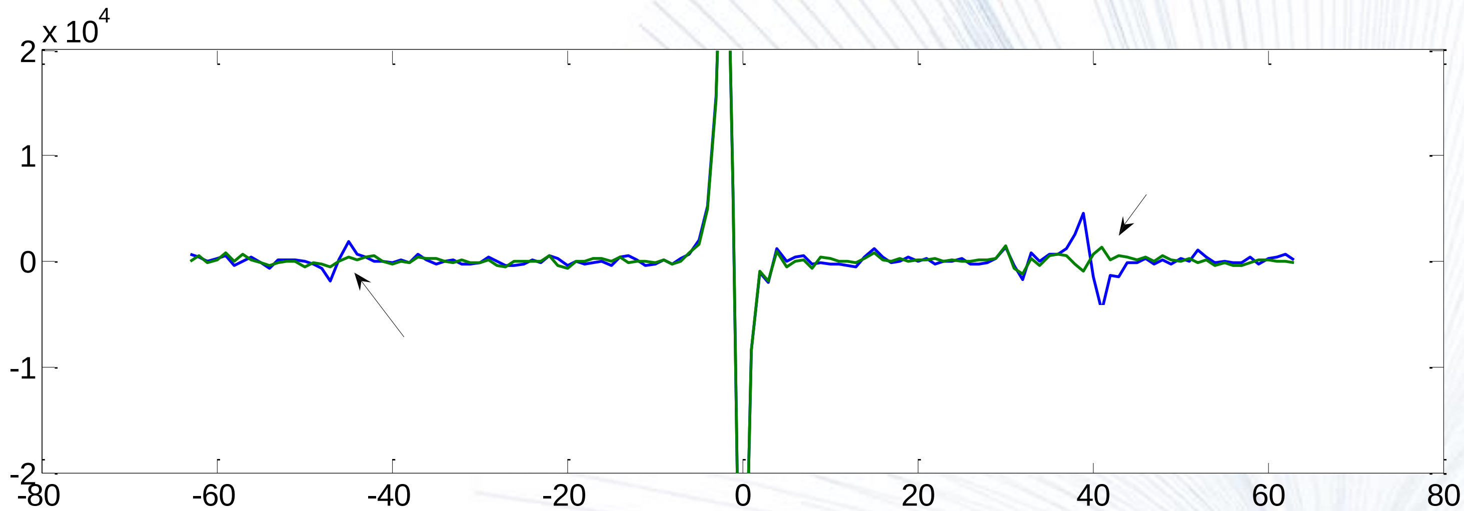
$$M(x,y) * R(x,y) = X(x,y)$$

Standard PRBS Testing Techniques in One Dimension



Typical Pseudorandom Sequence Test Results in One Dimension

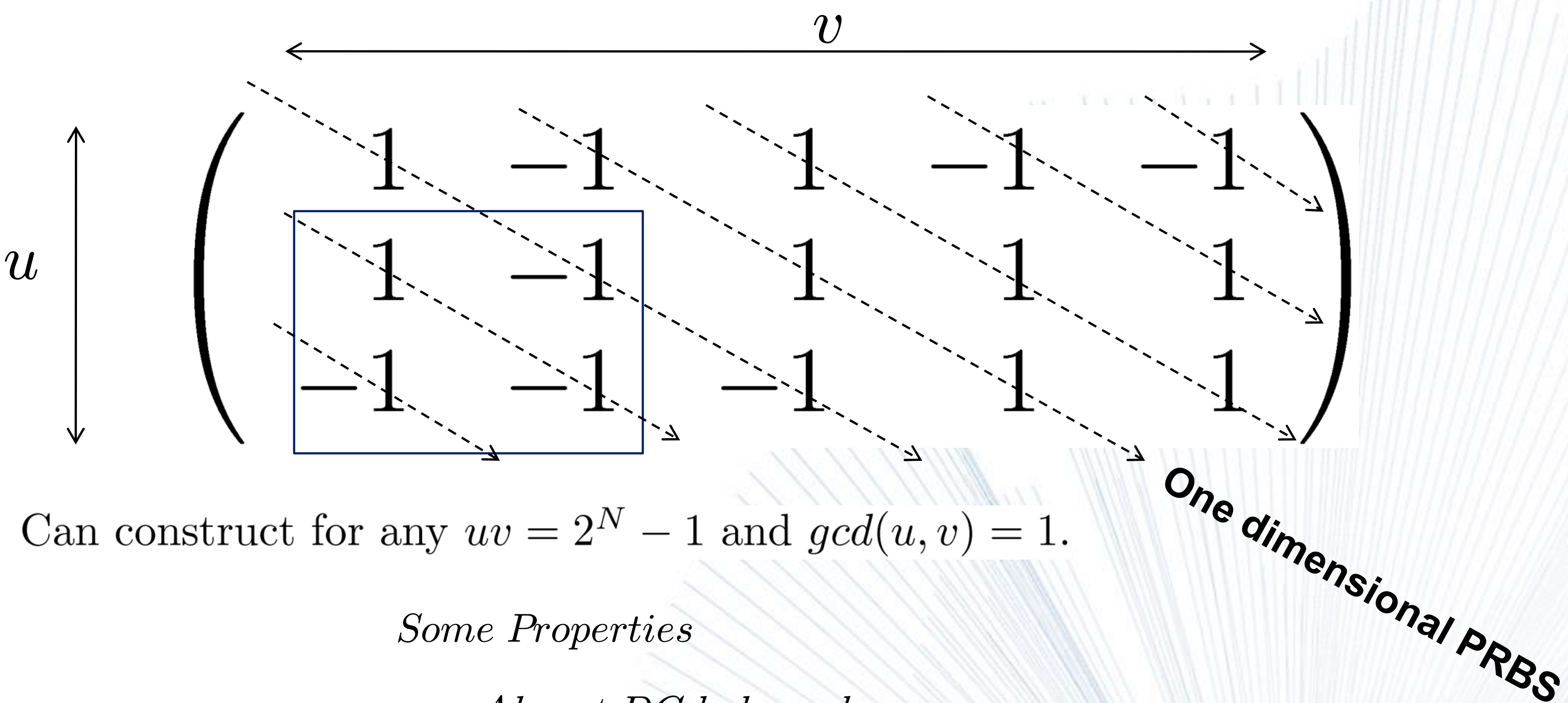




These results are LMR recording technology, not PMR

Suggested reading: Palmer et al, *IEEE Trans. on Magnetics,* Vol. 24, Issue 6, 1988
Hermann, *IEEE Trans. on Magnetics,* Vol. 26, Issue 5, 1990

Two Dimensional Pseudorandom Arrays



$$\begin{pmatrix} 1 & -1 & 1 & -1 & -1 \\ 1 & -1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 1 & 1 \end{pmatrix}$$

Can construct for any $uv = 2^N - 1$ and $gcd(u, v) = 1$.

Some Properties

- *Almost DC balanced*
- *2D window property*
- *Two-valued 2D autocorrelation*
- *Shift-multiply property*

Suggested reading: MacWilliams and Sloane, *Proc. of the IEEE,* Vol. 64, Num 12, December 1976
T. Etzion, *Trans. on Information Theory,* Vol. 34, No. 5, September 1988

A more practical example of a 2D PRBS

$$uv = 3 \times 85 = 255$$

$$\left(\begin{array}{ccccccccccccccccccccccccccccccccccccccccccc}
+ & + & - & - & + & - & - & - & - & + & + & - & - & + & + & + & + & - & - & + & - & - & + & + & - & + & + & - & + & + & + & + & + & - & - & - & - & - & - & - & + & + & + \\
+ & - & - & - & - & + & + & + & + & - & + & - & + & - & - & + & - & + & + & - & - & + & - & + & + & + & + & - & - & - & + & + & + & - & - & + & + & - & + & - & + & - & + \\
- & + & - & - & + & + & + & + & + & + & - & - & + & + & + & - & + & + & + & + & - & + & + & - & + & - & - & - & + & + & - & - & - & - & - & + & + & - & + & - & - & + & -
\end{array}\right.$$

$$\left.\begin{array}{cccccccccccccccccccccccccccccccccccccccccc}
- & + & - & - & - & - & - & + & - & + & - & - & + & + & - & - & - & + & + & - & + & + & + & - & - & + & - & + & - & - & - & - & + & - & + & + & + & - & + & + & - & - \\
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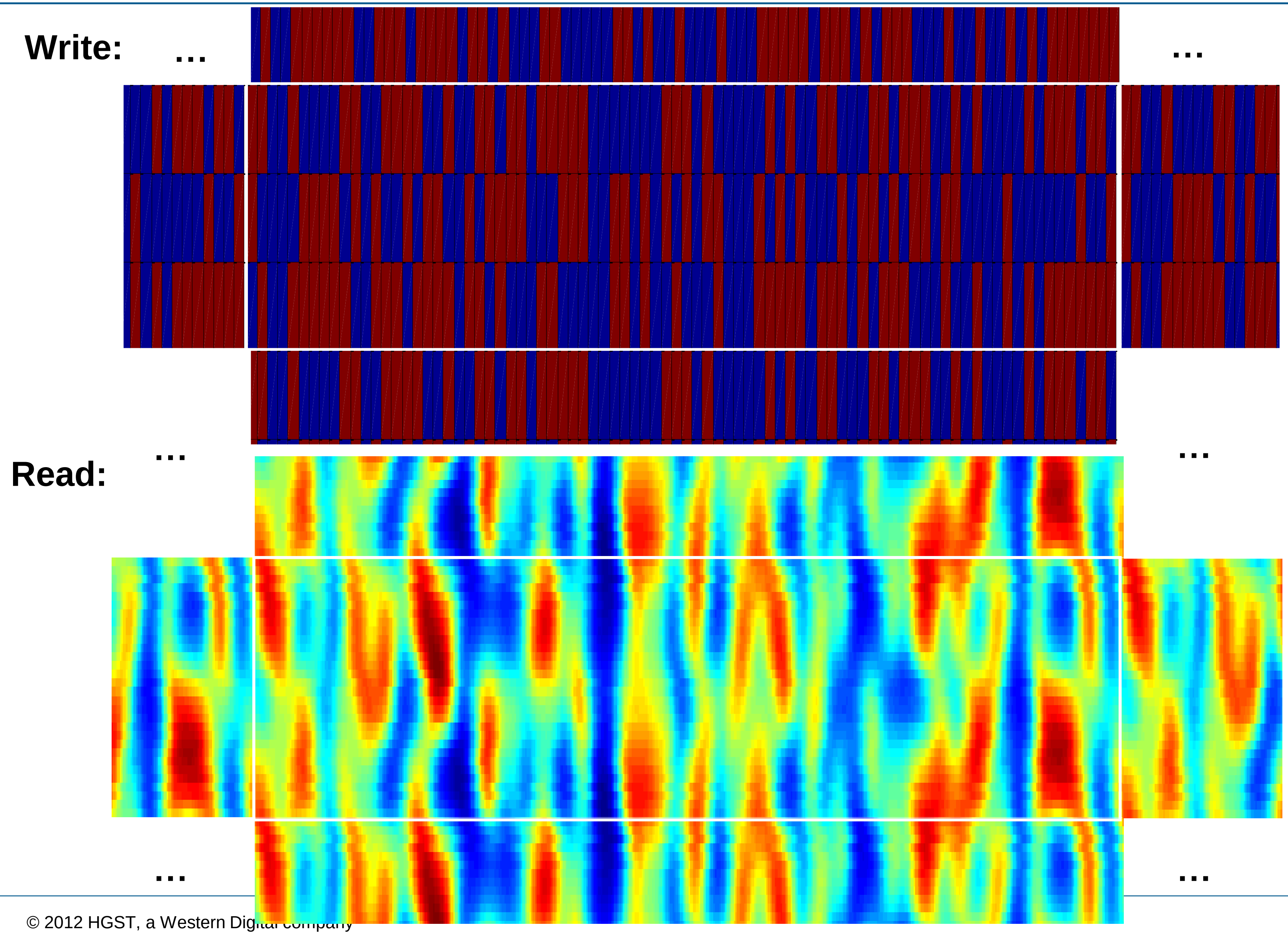
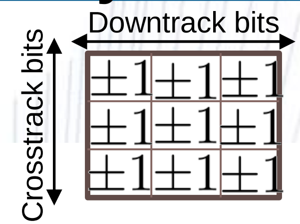
\end{array}\right)$$

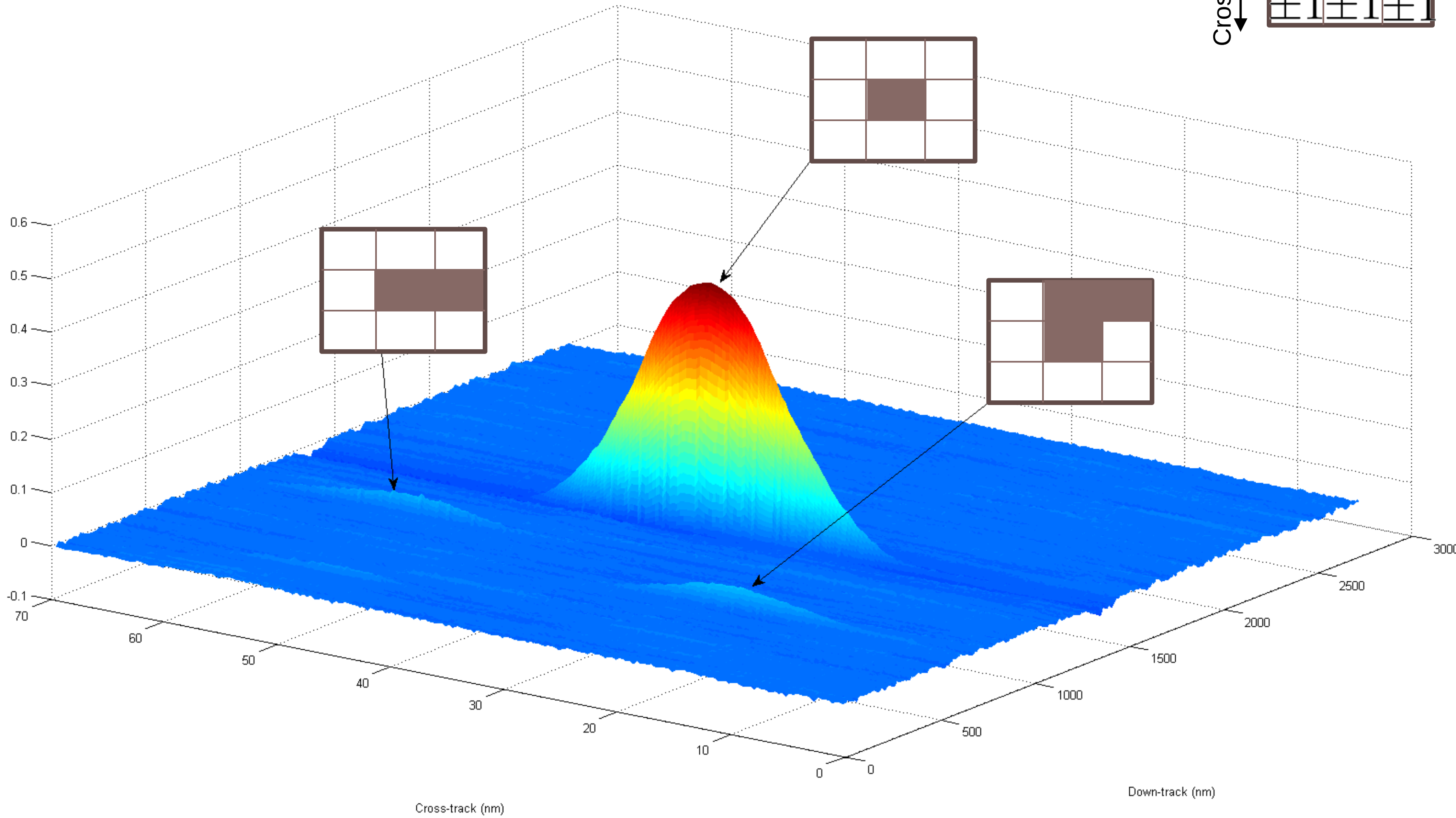
2D Array Experimental Method



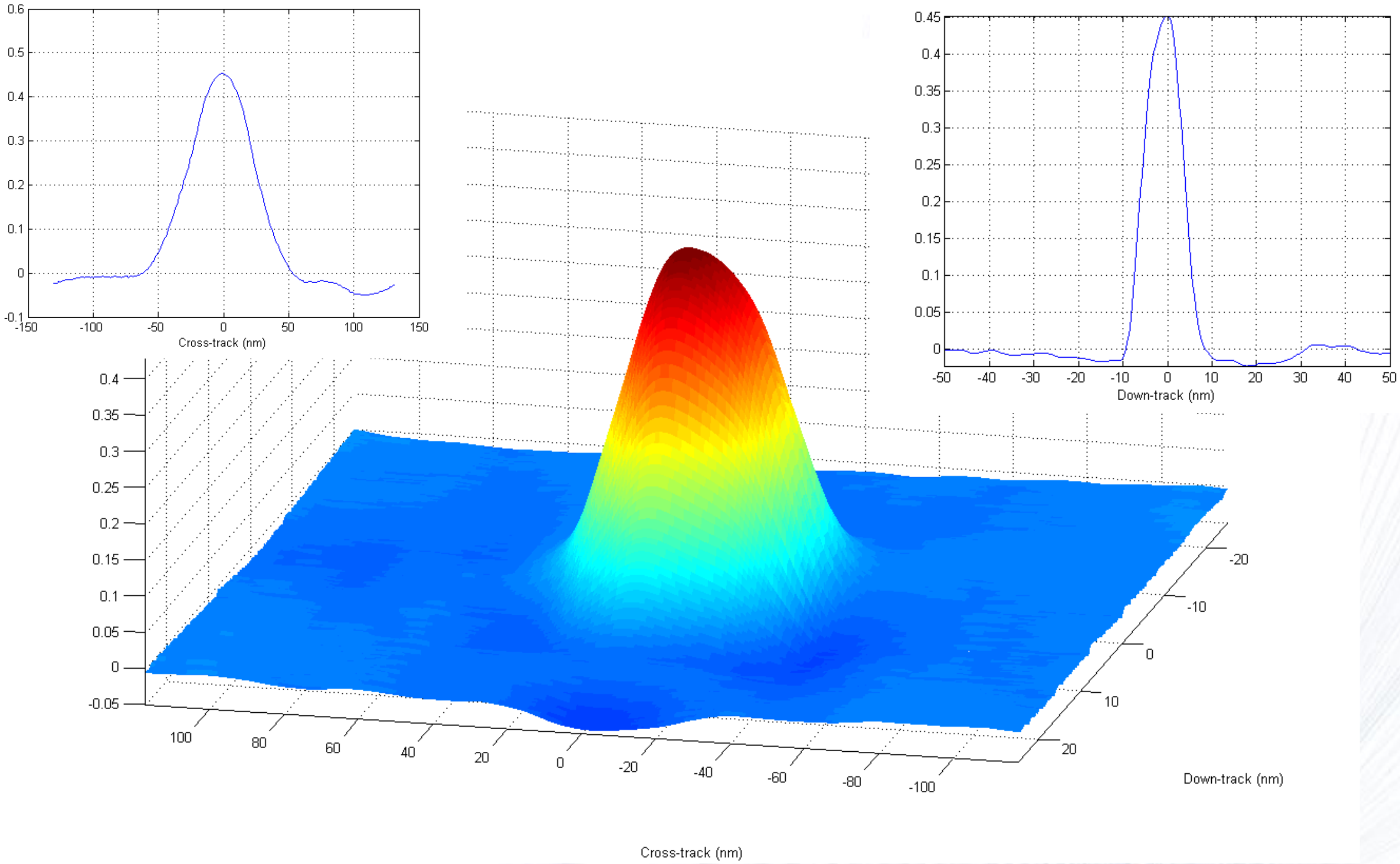
Measured 2D Deconvolution Result with Pseudorandom Array Pattern

Volterra kernels: local interaction map key:



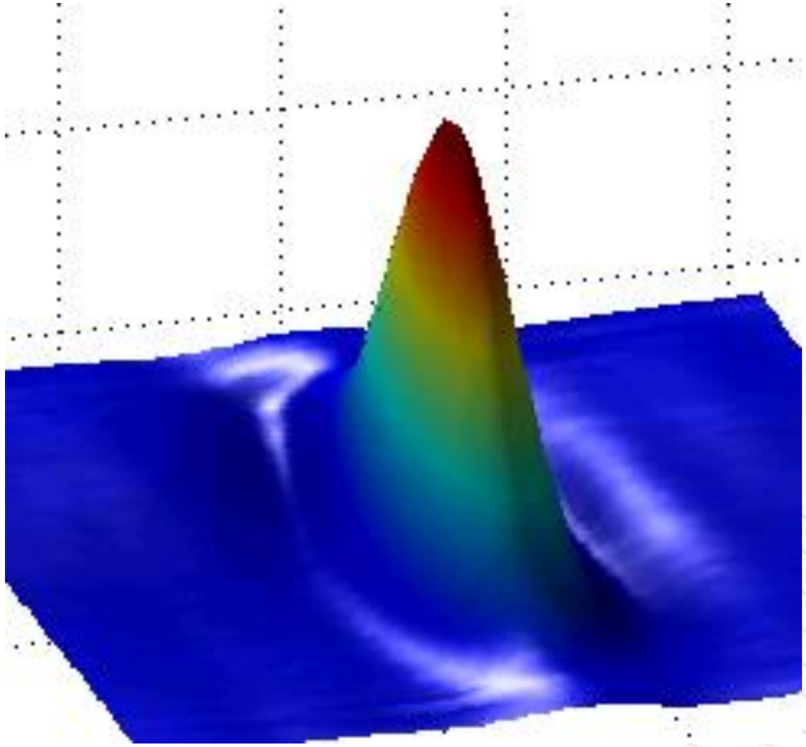


A Closer Look at the Measured "Patch Response" (the linear kernel)

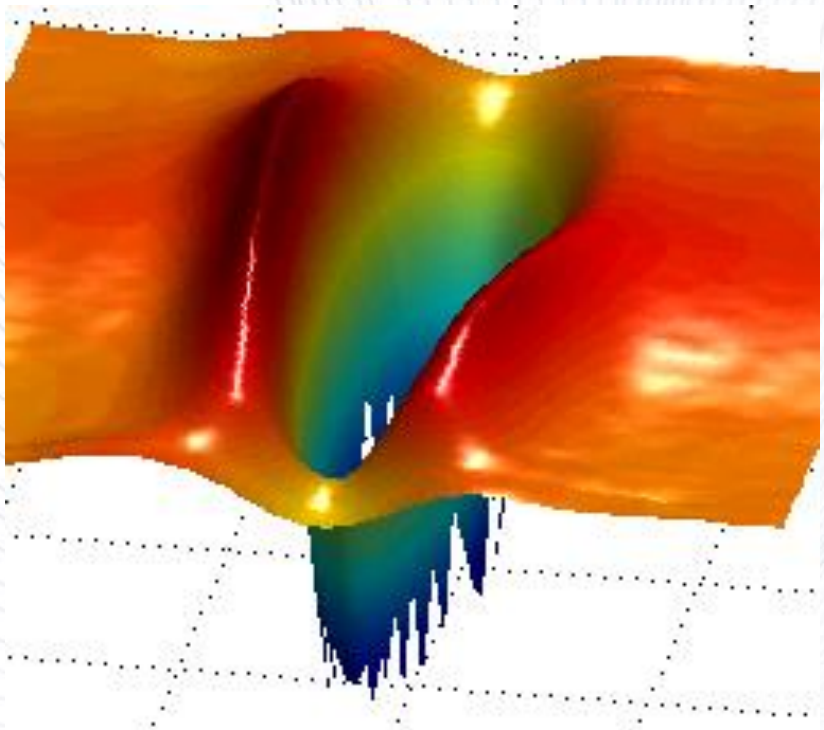


A second example of the linear portion of a 2D Pulse Response

Top View



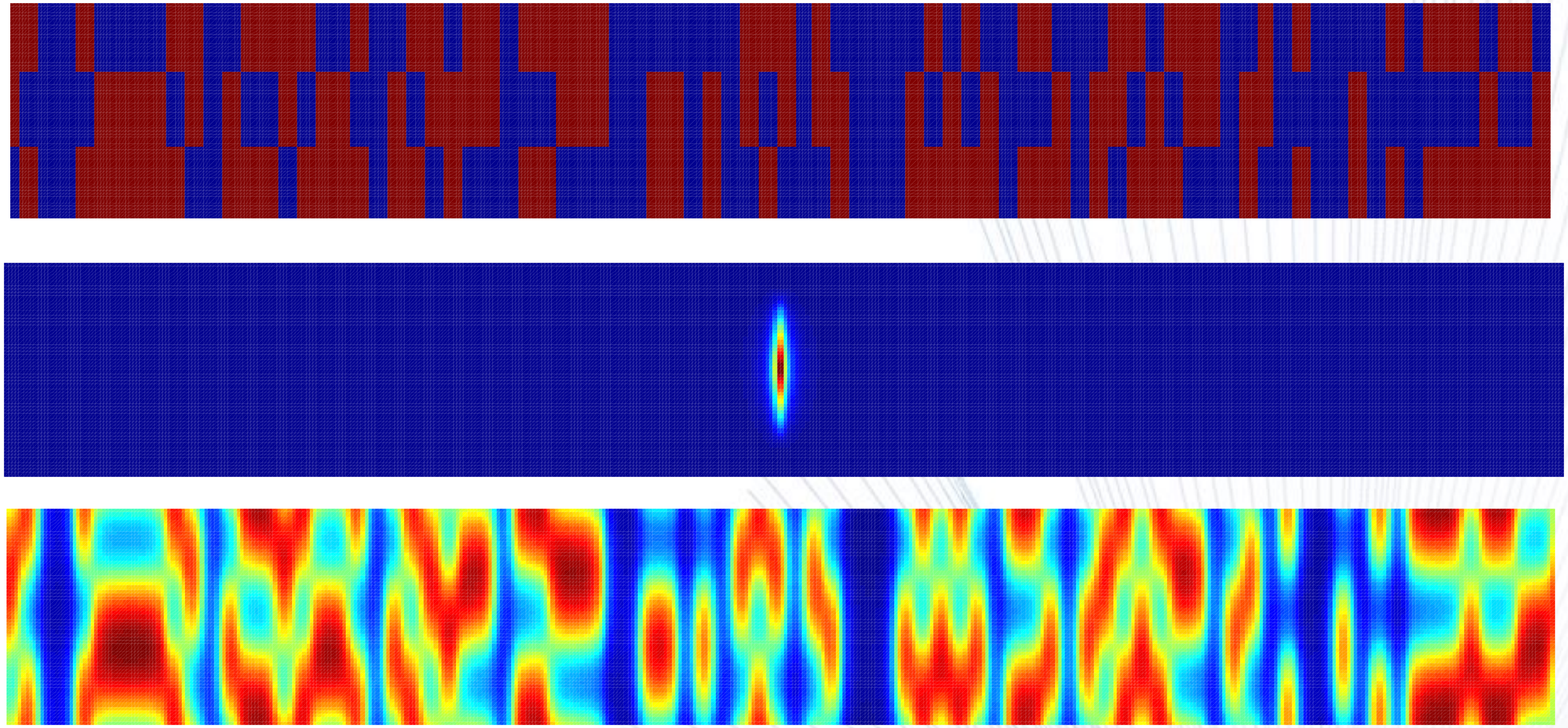
Bottom View



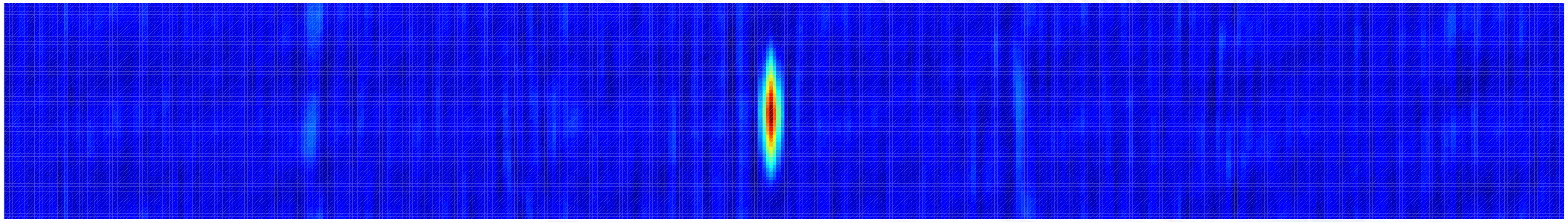
OK, what about the *Read Sensitivity Function?* *(a.k.a. 2D impulse response)*

$$M(x,y) * R(x,y) = X(x,y)$$

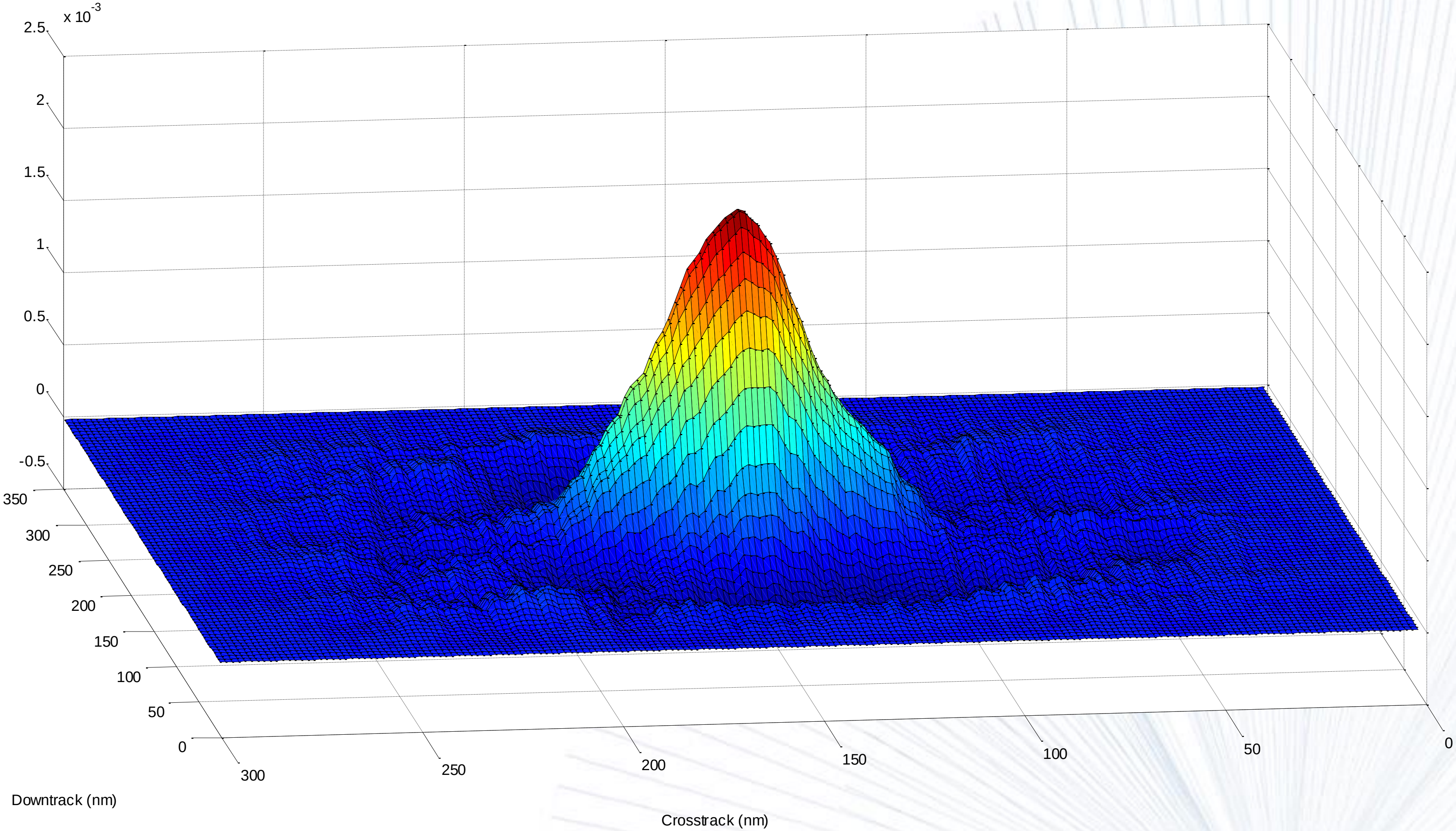
Simulated data



$$\hat{R} = X/\mathrm{sgn}(X)$$

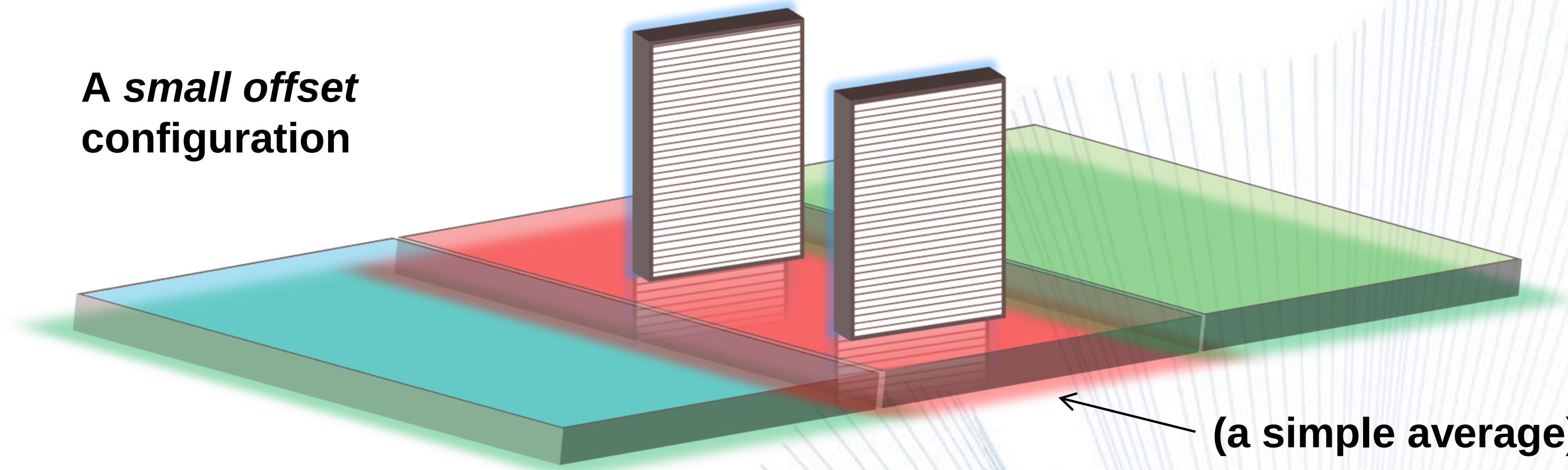


Measured 2D Read Sensitivity Function via the Projection Method



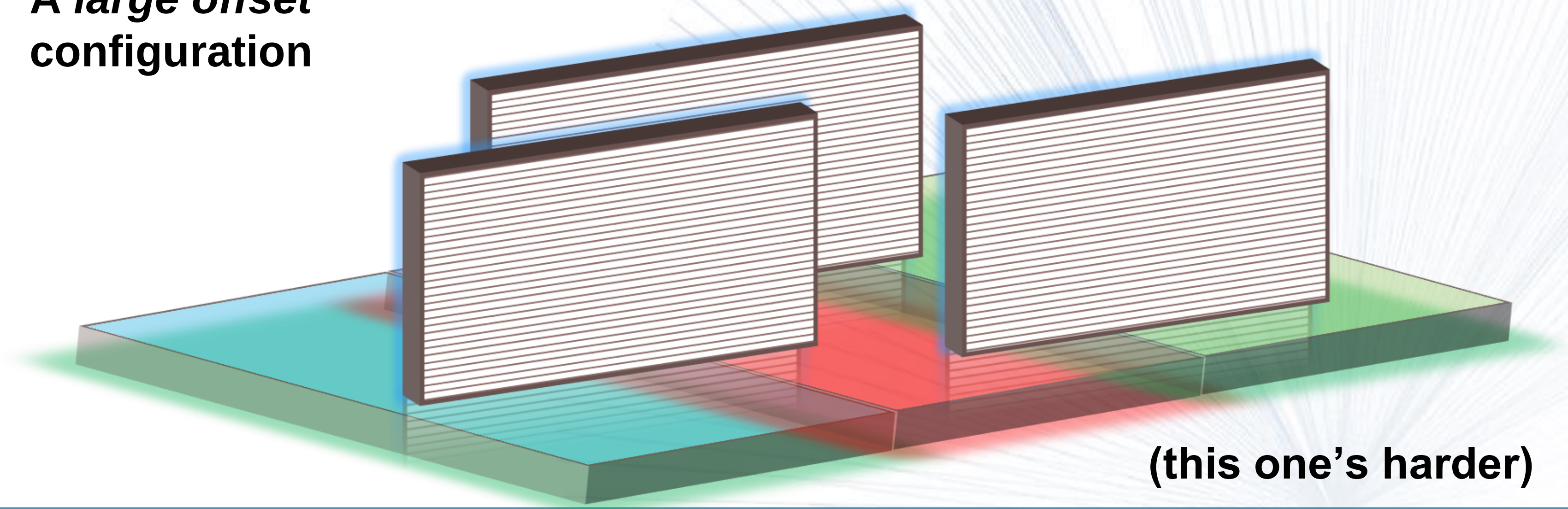
Two examples of TDMR Magnetic Systems and 2D Magnetic Signal Processing Configurations

A *small offset* configuration



(a simple average)

A *large offset* configuration



(this one's harder)

Analyzing the Small Offset Configuration

Assumption: **the two readers see identical magnetization but have independent "electronic" noises:**

$$X_1 = X_m + N_1 \qquad N_1 \sim \mathcal{N}(0, \sigma_1^2)$$

$$X_2 = X_m + N_2 \qquad N_2 \sim \mathcal{N}(0, \sigma_2^2)$$

Then, an optimal technique is to use this estimate of X_m **:**

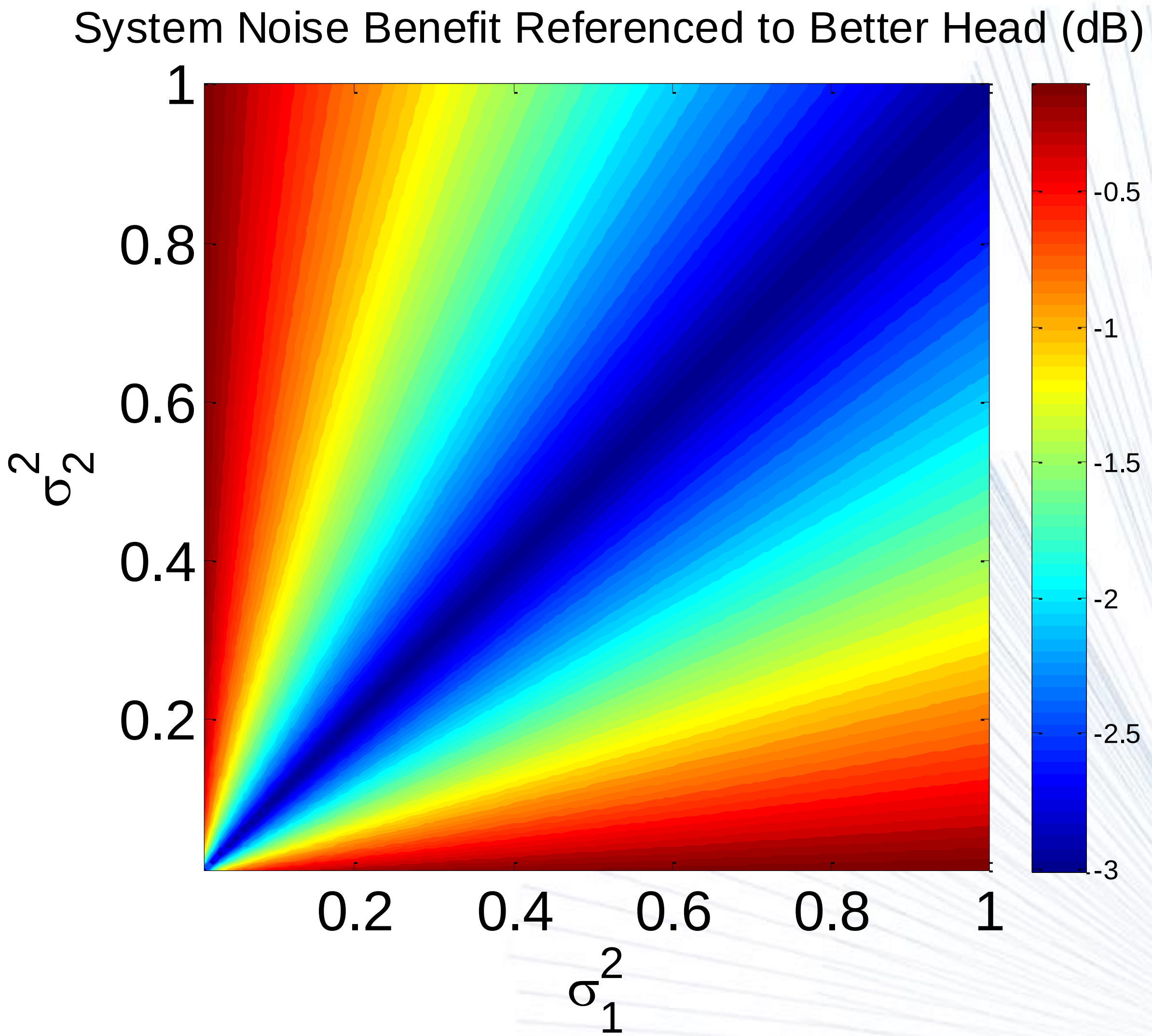
$$\hat{X}_m = \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2} X_1 + \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2} X_2$$

Note that this signal processing step can be written in matrix notation:

$$\hat{X}_m = \begin{pmatrix} \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2} & \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

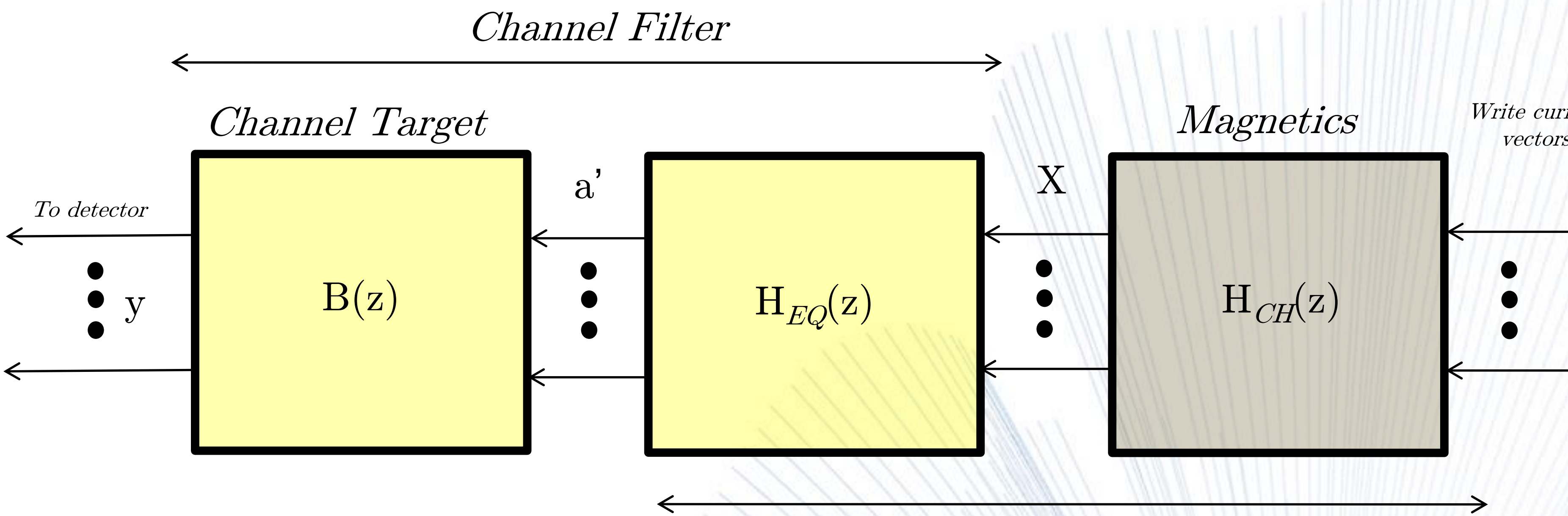
Suggested reading: H.L. Van Trees, "*Detection, Estimation, and Modulation Theory Part I*", any edition.

Small Offset Configuration Advantages



$$\sigma'^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

The Matrix Language of 2D Signal Processing

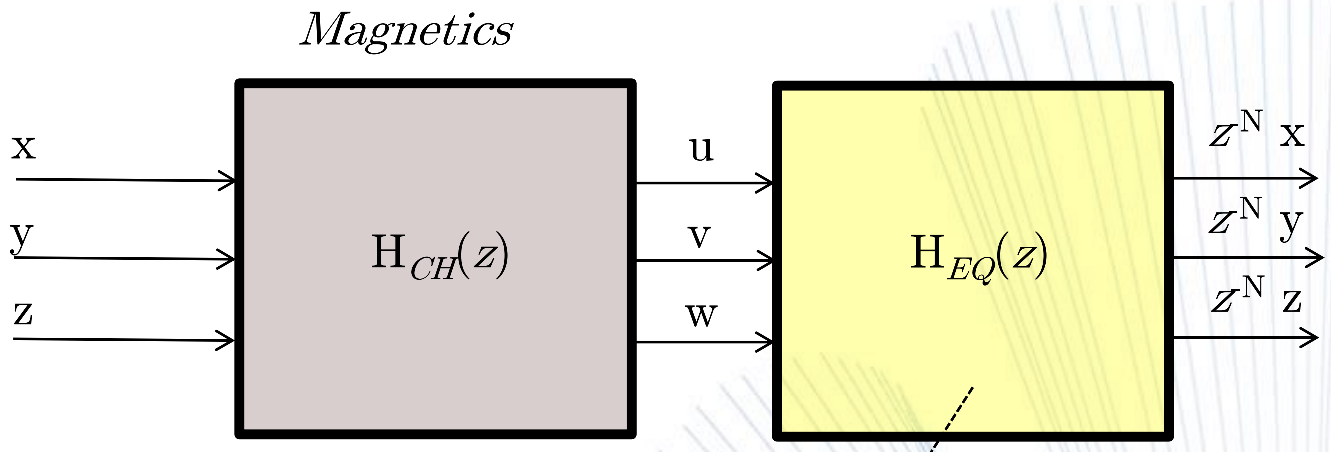


$$\mathbf{H}_{EQ}(z)\mathbf{H}_{CH}(z) \approx \mathbf{I}z^{-N}$$

$$\mathbf{y} = \mathbf{B}\mathbf{H}_{EQ}\mathbf{H}_{CH}\mathbf{a} \approx \mathbf{B}\mathbf{a}$$

Suggested reading: P. Vaidyanathan, "*Multirate Systems and Filter Banks*", 1992

3x3 equalization design equations

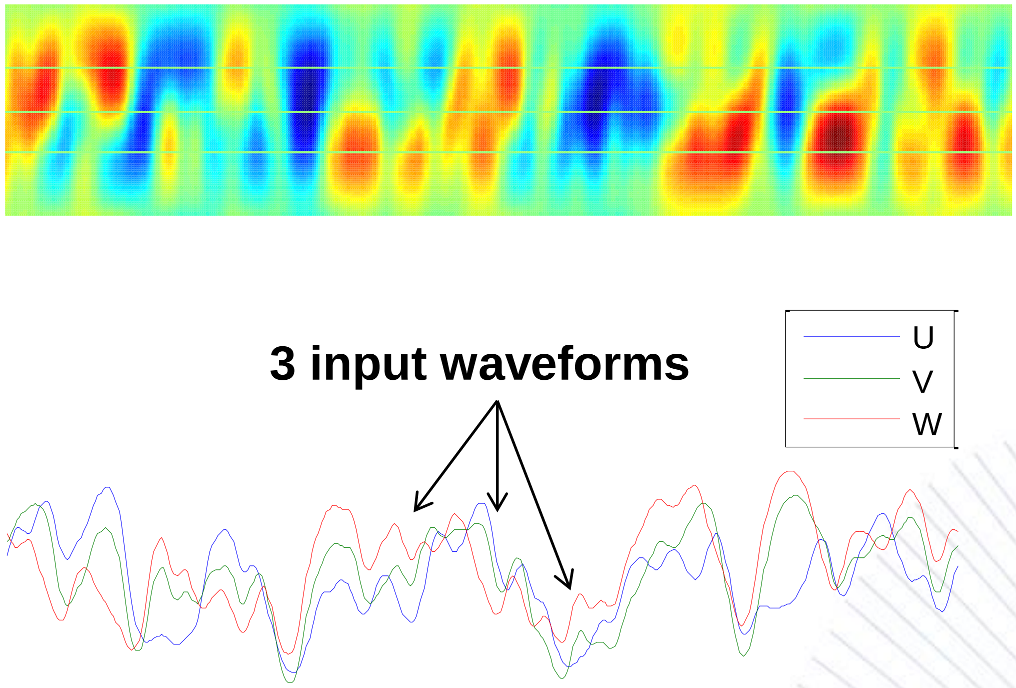


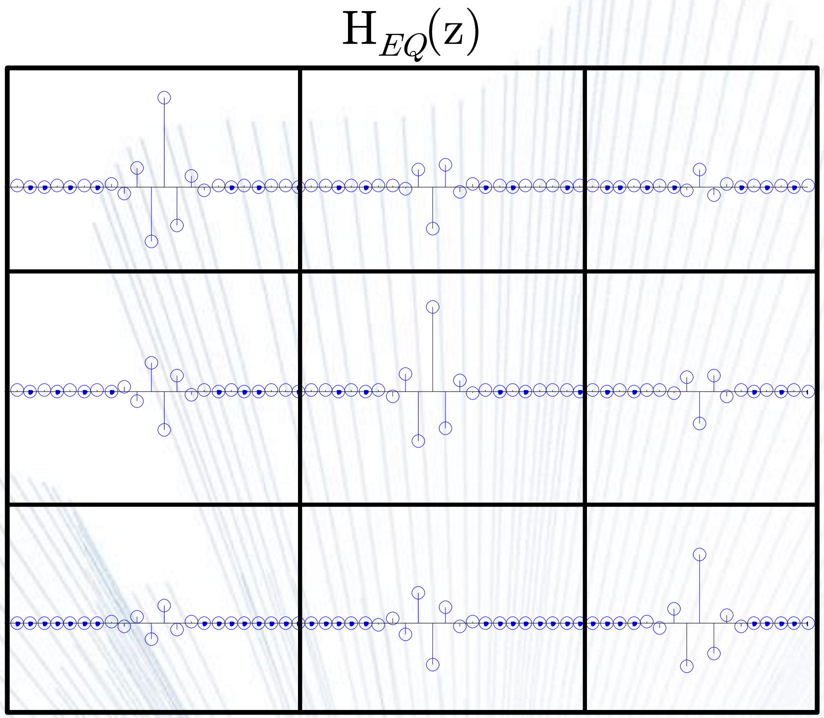
3 x Ntaps

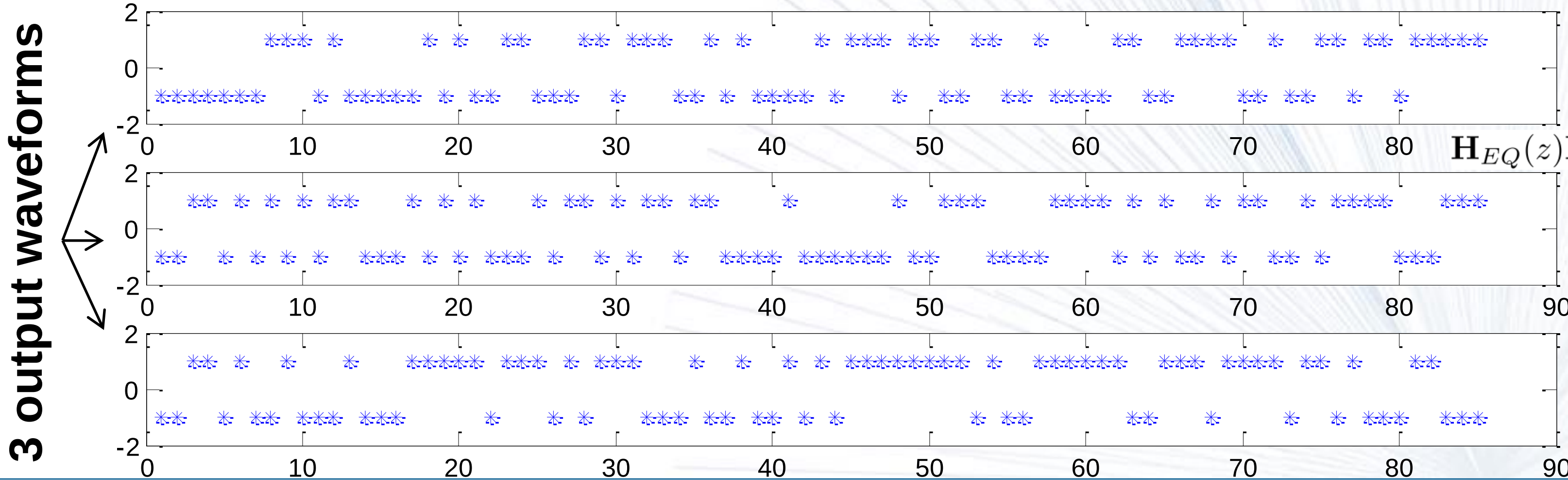
$$\begin{pmatrix} \mathbf{R}_{uu} & \mathbf{R}_{uv} & \mathbf{R}_{uw} \\ \mathbf{R}_{vu} & \mathbf{R}_{vv} & \mathbf{R}_{vw} \\ \mathbf{R}_{wu} & \mathbf{R}_{wv} & \mathbf{R}_{ww} \end{pmatrix} \begin{pmatrix} \mathbf{h}_{ux} & \mathbf{h}_{vx} & \mathbf{h}_{wx} \\ \mathbf{h}_{uy} & \mathbf{h}_{vy} & \mathbf{h}_{wy} \\ \mathbf{h}_{uz} & \mathbf{h}_{vz} & \mathbf{h}_{wz} \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{xu} & \mathbf{r}_{yu} & \mathbf{r}_{zu} \\ \mathbf{r}_{xv} & \mathbf{r}_{yv} & \mathbf{r}_{zv} \\ \mathbf{r}_{xw} & \mathbf{r}_{yw} & \mathbf{r}_{zw} \end{pmatrix}$$

3 x Ntaps (rows of the R matrix); 3 (columns of h); 3 (columns of r); 3 x Ntaps (rows of r)

Inverting the Magnetic Channel

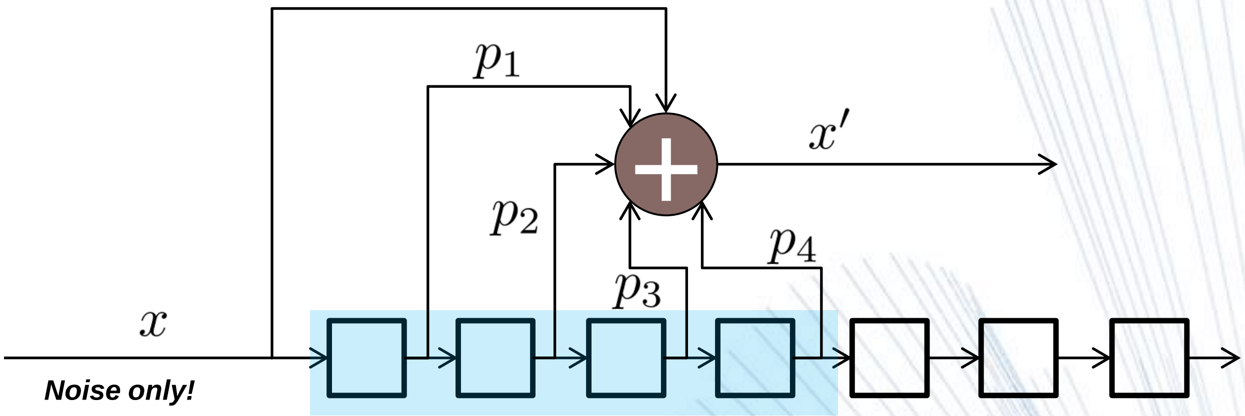






$$\mathbf{H}_{EQ}(z)\mathbf{H}_{CH}(z) \approx \mathbf{I}z^{-N}$$

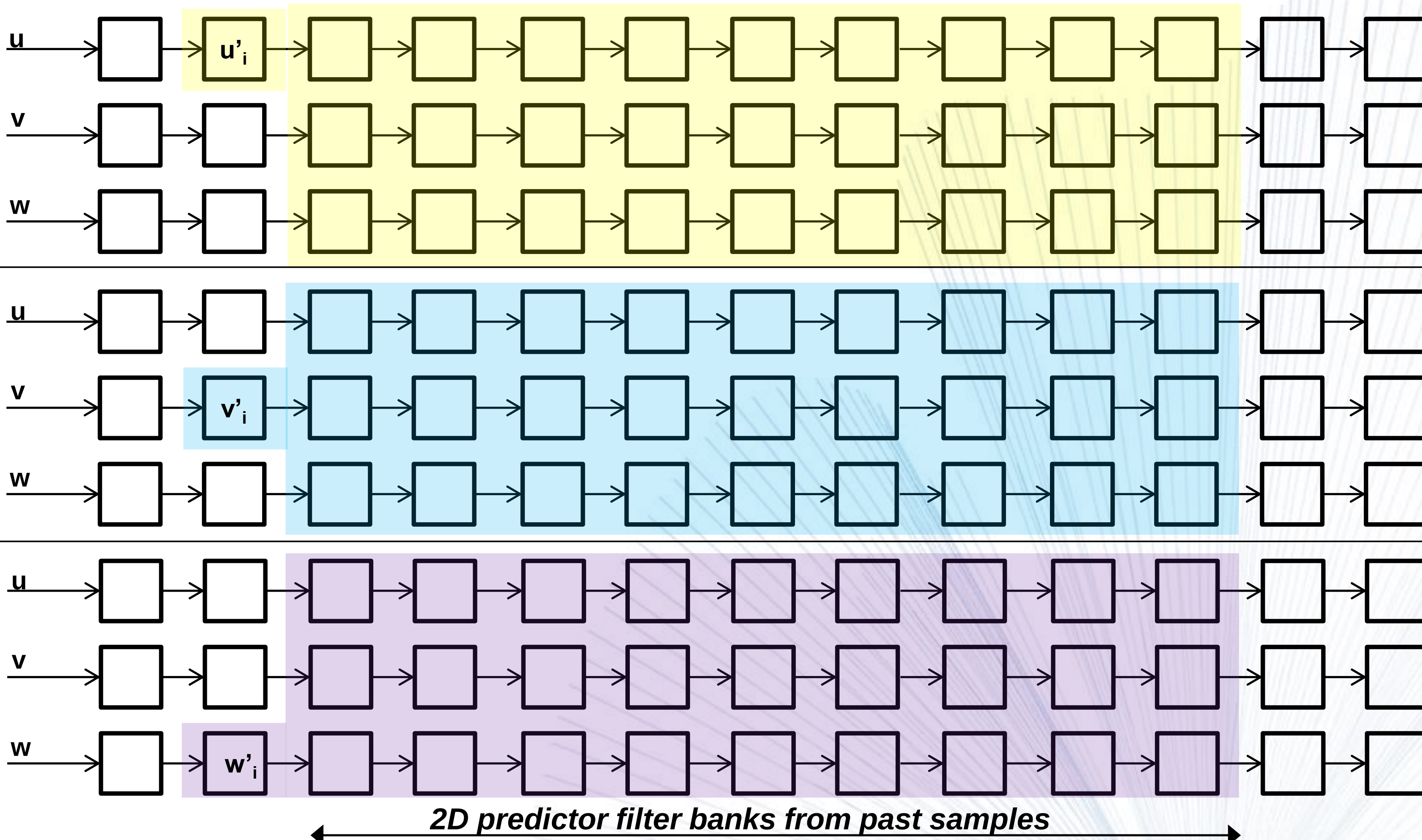
Noise Prediction and Minimization from Past Samples in One Dimension



$$B(z^{-1}) = 1 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3} + p_4 z^{-4} + \ldots$$

Suggested reading: Coker et al, *IEEE Trans. on Magnetics,* Vol. 34 p 110-177, 1998

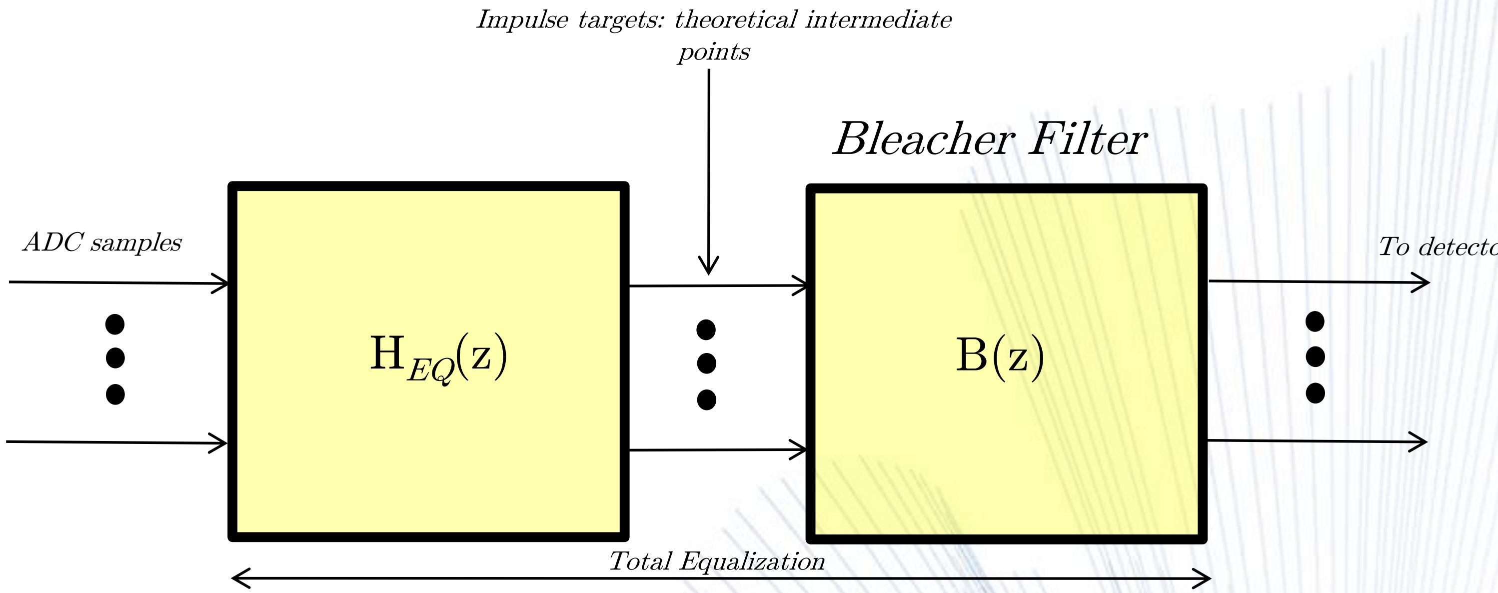
Two Dimensional Noise Prediction and Minimization



Thm: **u', v', and w' are mutually orthogonal (uncorrelated) everywhere when power-minimized, *except* when aligned in the same time instant.**

Key assumption: WSS noise

Design equations for 2D noise prediction from *past* samples based on noise power minimization/whitening



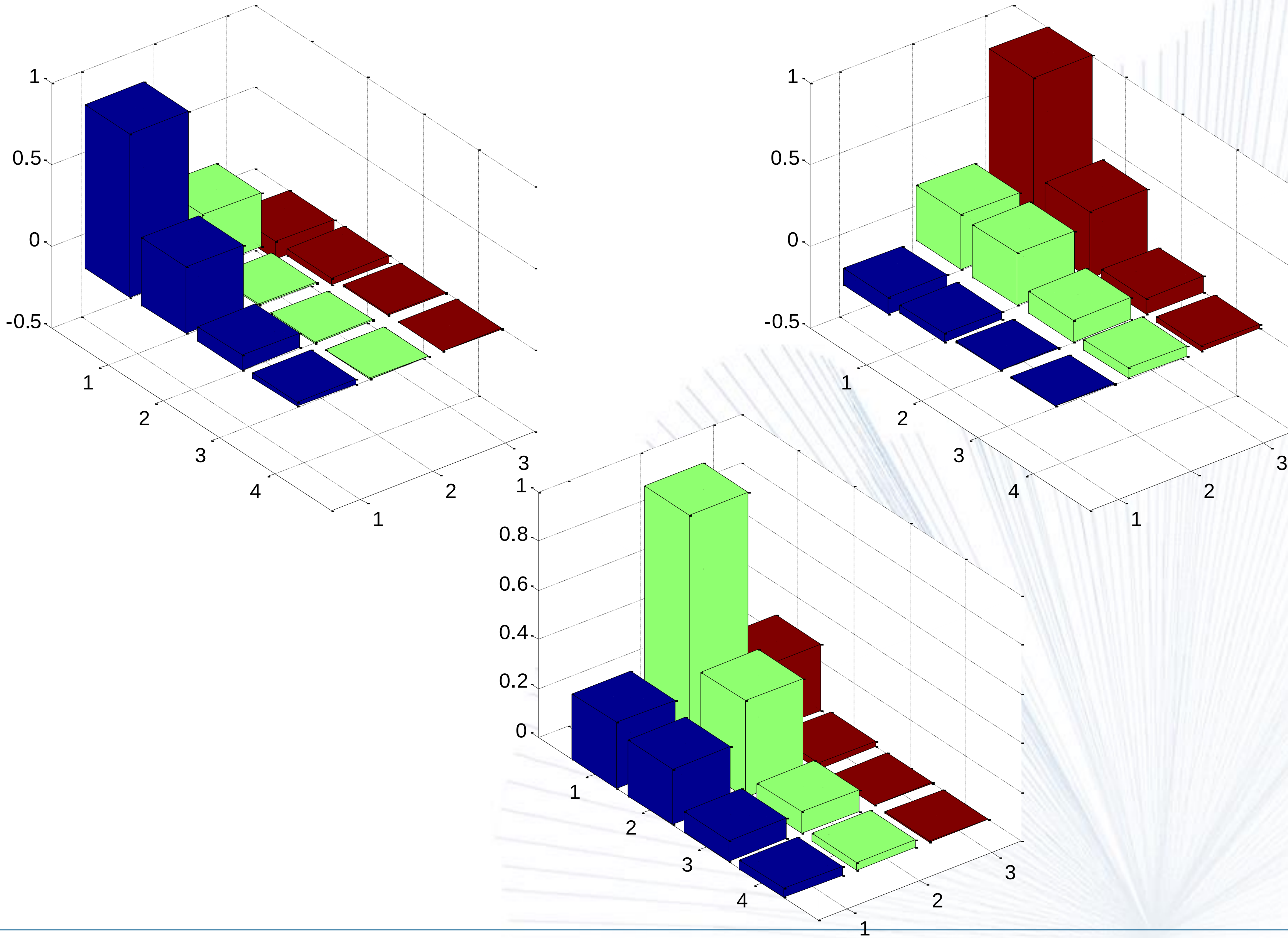
3x3 form example for minimization filter:

$$\mathbf{B}(z) = \begin{pmatrix} 1 + p_{11}z^{-1} + p_{12}z^{-2} + ... & q_{11}z^{-1} + q_{12}z^{-2} + ... & r_{11}z^{-1} + r_{12}z^{-2}... \\ p_{21}z^{-1} + p_{22}z^{-2} + ... & 1 + q_{21}z^{-1} + q_{22}z^{-2} + ... & r_{21}z^{-1} + r_{22}z^{-2}... \\ p_{31}z^{-1} + p_{32}z^{-2} + ... & q_{31}z^{-1} + q_{32}z^{-2} + ... & 1 + r_{31}z^{-1} + r_{32}z^{-2}... \end{pmatrix}$$

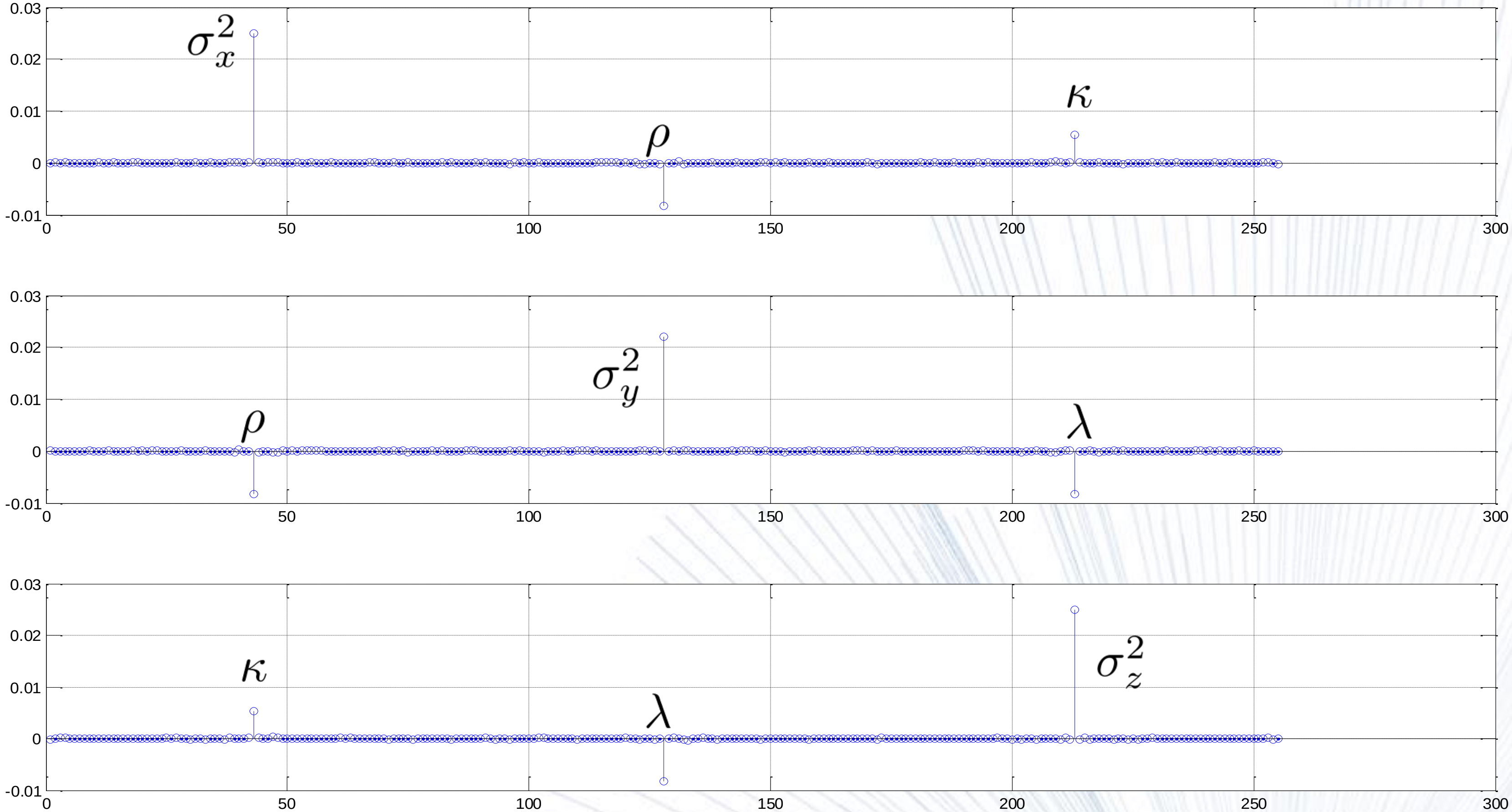
3x3 Design equations:

$$\begin{pmatrix} \mathbf{R}_{uu} & \mathbf{R}_{uv} & \mathbf{R}_{uw} \\ \mathbf{R}_{vu} & \mathbf{R}_{vv} & \mathbf{R}_{vw} \\ \mathbf{R}_{wu} & \mathbf{R}_{wv} & \mathbf{R}_{ww} \end{pmatrix} \begin{pmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 \\ \mathbf{q}_1 & \mathbf{q}_2 & \mathbf{q}_3 \\ \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 \end{pmatrix} = - \begin{pmatrix} \mathbf{r}_{uu}(1:N) & \mathbf{r}_{vu}(1:N) & \mathbf{r}_{wu}(1:N) \\ \mathbf{r}_{vu}(1:N) & \mathbf{r}_{vv}(1:N) & \mathbf{r}_{wv}(1:N) \\ \mathbf{r}_{wu}(1:N) & \mathbf{r}_{vw}(1:N) & \mathbf{r}_{ww}(1:N) \end{pmatrix}$$

An Example Detector Target Filter B



Noise Correlation Example after Noise Prediction with Past-Sample minimization



$$K = \begin{pmatrix} \sigma_x^2 & \rho & \kappa \\ \rho & \sigma_y^2 & \lambda \\ \kappa & \lambda & \sigma_z^2 \end{pmatrix}$$

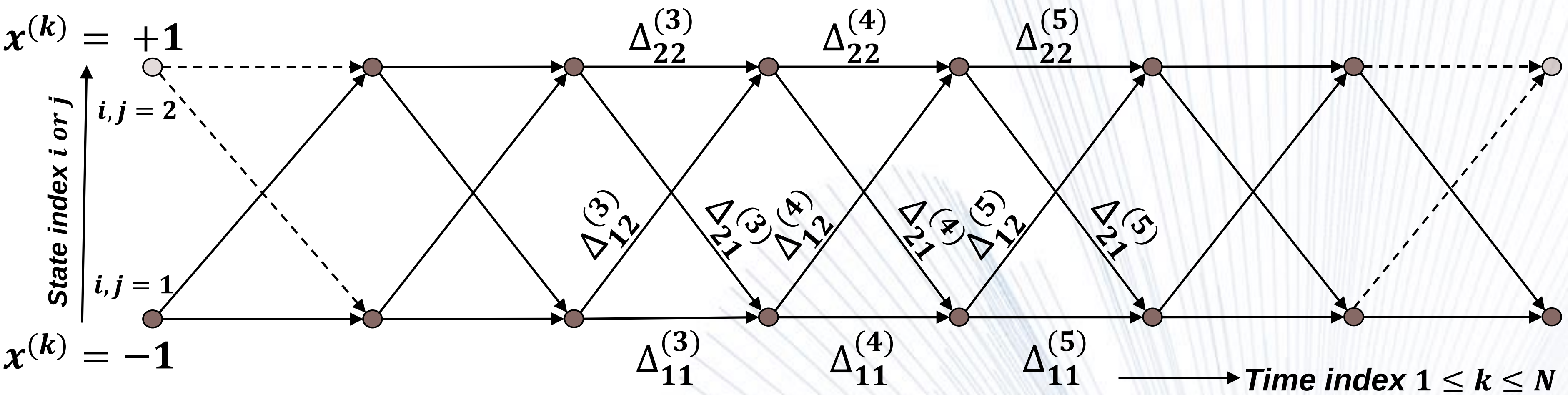
Maximum-Likelihood Detection of a Single Stream

"Noise" in the branch from state i to state j in the k^{th} instant.

Expected observation on the branch from state i to state j during the k^{th} instant. A function of a presumed input sequence x_0^N

$$\Delta_{ij}^{(k)} = Y_{ij}^{(k)} - y^{(k)}$$

Actual observation in the k^{th} instant.



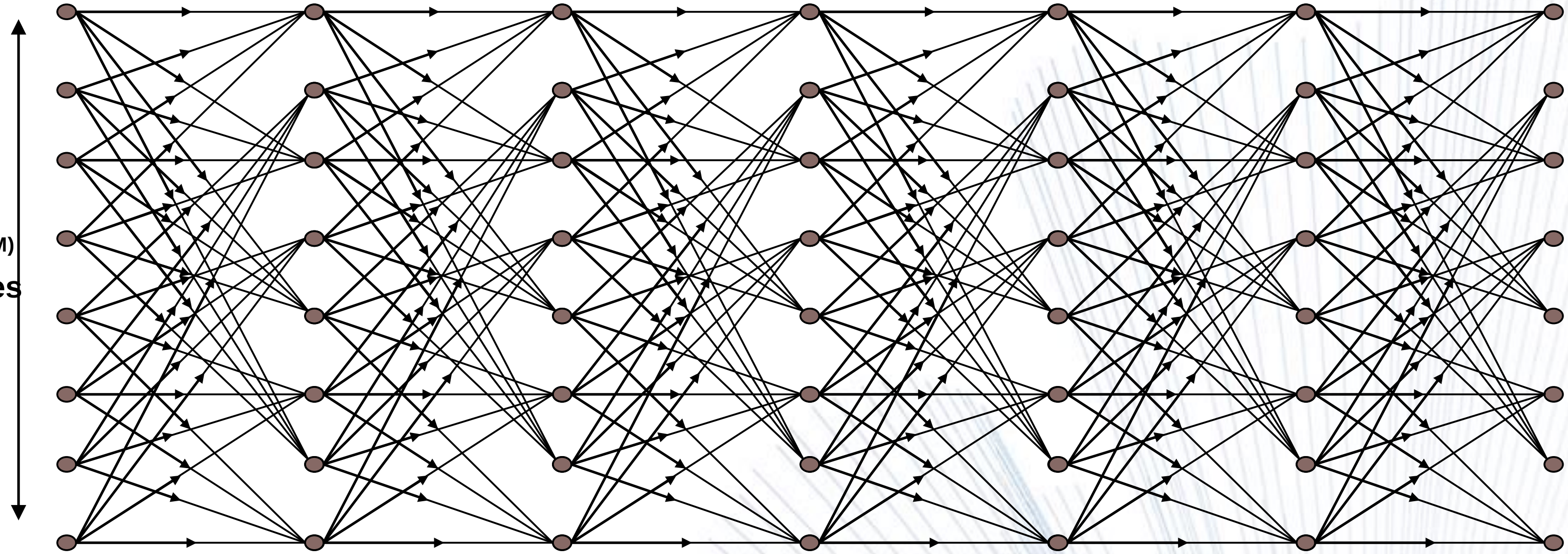
ML Sequence Detection: maximize

$$P\left(x_0^N \mid y_1^N\right) \propto \prod_k \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\Delta_{ij}^{(k)}\right)^2 / 2\sigma^2}$$

over all possible input sequences x_0^N.

Maximum-Likelihood for Joint Vector Input/Output Detection

Key additional assumption: Gaussian noise



T **is the total channel length = maximum span of nonzero taps relating all inputs to any output, and** ***M*** **is the number of channels.**

Key relationship: the branch probability

$$p(\Delta_{ij}) = \frac{1}{\sqrt{(2\pi)^M |K|}} e^{-\frac{1}{2}\Delta_{ij}^T K^{-1} \Delta_{ij}}$$

$$K \triangleq \mathrm{E}\{\Delta\Delta^T\}$$

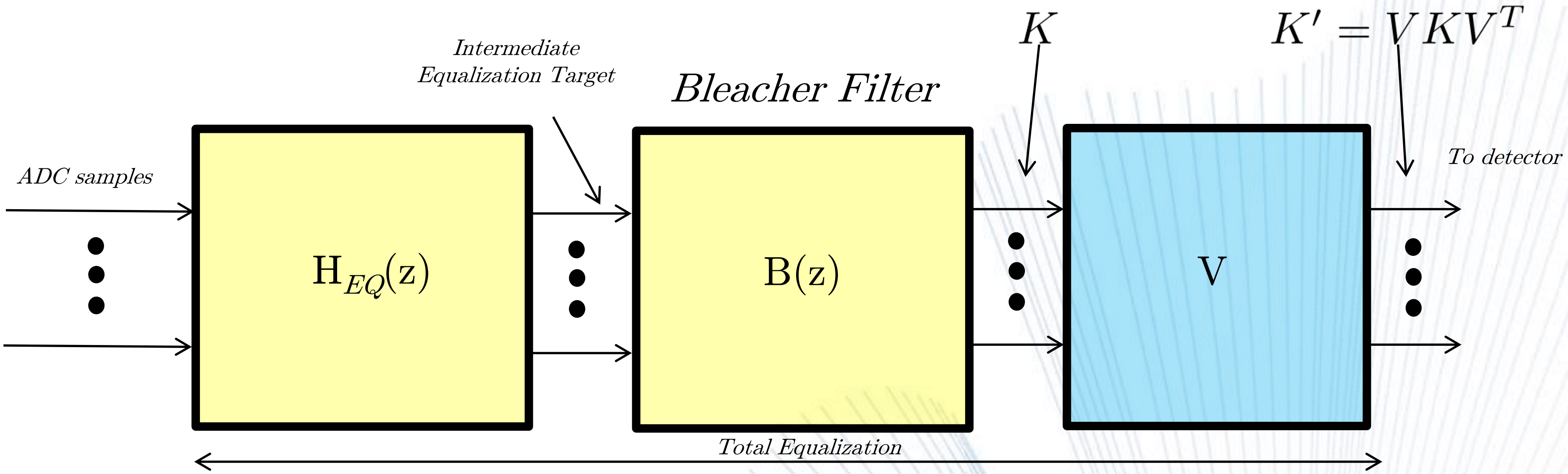
Example:

$$\Delta^T K^{-1} \Delta = (\Delta x \ \ \Delta y) \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}^{-1} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

$$= \frac{\Delta x^2 - 2\Delta x \Delta y \rho + \Delta y^2}{1-\rho^2}$$

Suggested reading: MacKay, "Information Theory, Inference, and Learning Algorithms", Cambridge, 2003

New 2D effect: Invariance after invertible matrix rotation



$$p(\Delta_{ij}) = \frac{1}{\sqrt{(2\pi)^M |K'|}} e^{-\frac{1}{2}\Delta_{ij}^T K'^{-1} \Delta_{ij}}$$

DOES NOT depend on the details of V!

Special case: eigendecomposition of $K = V_E^T \Lambda V_E$ gives diagonal form $K'^{-1} = \Lambda^{-1}$

TDMR Talk Summary

- Introduction
- Magnetic Models in Two Dimensions
- Testing in Two Dimensions
- Magnetic Signal Processing in Two Dimensions
- Questions

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